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- chapter 1 -



MS1: Advanced finite element technology for discontinuities and evolving interfaces



# Numerical modelling of load-sequence effects on fatigue crack growth

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Fatigue is one of the main drivers in the design process of offshore wind structures. Steel jacket or monopile structures suffer from cyclic loads, originating from coupled wind and wave loads acting on the structure. Due to the stochastic nature of these loads, wind turbines are subjected to variable amplitude loads. It is known that load-sequence can cause retardation or acceleration of fatigue crack growth. In fact, an overload can cause retardation, often attributed to the plasticity around the crack tip and wake.

To model this counterintuitive phenomenon a cohesive zone is typically used in combination with a plasticity model. However, these models often lack physical understanding and generality, therefore requiring many curve fitting parameters.

In this study a new method has been developed for modelling load-sequence effects on fatigue crack growth of ductile materials. A cycle-by-cycle approach is adopted to capture the plasticity around the crack tip. The cyclic plasticity model [1] is combined with a novel crack propagation method, which requires a limited number of fitting parameters and is based on combining fracture and damage mechanics. Local fracture parameters are converted to global entities inspired by the Thick Level Set method [2], which in turn are used to determine the incremental crack length. In this approach, damage attributed to cyclic loading is directly related to Paris Law parameters. The method is compared to experiments performed on four-point bending specimens subjected to various overloads.

- [1] J. Lemaitre and J.L. Chaboche, Mechanics of Solid Materials. *Cambridge University Press*, Cambridge, 1990.
- [2] N. Moes, C. Stolz, P.E. Bernard and N. Chevaugeon, A level set based model for damage growth: The thick level set approach. *International Journal for Numerical Methods in Engineering*, 86(3), 358-380, 2011.

# Finite element interface model for simulating delamination using a thick level set approach

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This paper presents a damage model for simulating delamination in composites. To predict delamination, interface elements are commonly used to model the discontinuity between layers of a laminate. In conventional formulations of these elements, the damage variable is a function of the displacement jump. However, in the current application following the thick level set approach [1,2], a band of damage with a predefined length is considered, inside which damage is a function of the distance to a front that is defined with a level set function.

In the proposed model, this definition of damage is embedded in the constitutive relationship of interface elements. The crack front and the band of damage are updated based on the configurational force for crack growth. This driving force is computed from integrating local driving forces across the damaged band. In contrast with the earlier continuum damage models in the context of the thick level set method [1,2], the damage is now applied to an initially stiff interface and the configurational force is computed from the displacement jump.

This paper investigates the specification of a suitable damage function for the damage distribution in this approach. A 3D model is developed, and the numerical results obtained from double cantilever beam simulations are critically assessed.

- [1] N. Möes, C. Stolz, P.-E. Bernard. A level set based model for damage growth : the thick level set approach. International Journal For Numerical Methods In Engineering, 86(3), 358-380, 2011.
- [2] F.P. van der Meer, L.J. Sluys. The Thick Level Set method: sliding deformations and damage initiation. Computer Methods in Applied Mechanics and Engineering; 285, 64-82, 2015.

# Advanced finite elements to simulate the damage process in ceramic-matrix composites

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The introduction of ceramic-matrix composites (CMCs) into hot parts of aircraft engines is an important topic in the aeronautics field. Three-dimensional woven SiC/SiC CMCs are considered as serious candidates to replace metals in high-temperature structures such as turbine blades or exhaust nozzles. CMCs are made of three basic constituents: ceramic fibers and matrix, and fiber/matrix interfaces. Even if their constituents are brittle, CMCs exhibit a quasi-brittle behaviour. Indeed, they experience multiple cracking associated with fiber/matrix interfacial debonding which leads to a damage-tolerant behavior [1].

Because of their complex multiscale architecture, predictive simulations of their behavior is a challenging task. Thus, current design methods of CMCs parts still vastly rely on experiments and empirical considerations, which hinders the use of CMCs at their full potential. A reliable modeling tool would greatly help material developers to gain a better insight on how the composite architecture and constituents impact the resulting macroscopic behavior. Another specific issue related to CMCs is the need to obtain a precise estimate of the cracks densitiy and opening since it is a key factor in the estimation of their lifetime in oxidizing environments.

The development of such a numerical tool faces numerous challenges:

- a realistic representation of woven CMCs leads to large finite element models with several millions degrees of freedom
- the matrix crack density is considerable (>  $1mm^{-1}$  in the yarns direction [2]) and needs to be represented accurately
- the different modeling scales are not well separated which greatly challenges classical homogenization theories and requires the development of *ad-hoc* numerical bridges between the scales

The aim of this work is to propose a step towards a multiscale approach dedicated to the simulation of the damageable behavior of CMCs. To be able to represent the discrete cracks while keeping the cost of the simulation acceptable, advanced finite elements must be used. Matrix cracking is simulated using an embedded finite element based on [3] and [4]. The extra degrees of freedom are condensed at the element level which leads to an

entirely local formulation and avoids the use of costly tracking algorithms. Discrete damage zone models based on [5] are used to deal with interfacial debonding as they where proven to be numerically more efficient than classical cohesive zone models.

Preliminary simulations of damage process will be presented for a three-dimensionnal woven SiC/SiC composite, and computed crack density and opening will be discussed. A special attention will also be drawn to computation times, numerical robustness and mesh dependencies issues.

- [1] Guillaumat L & Lamon J. Probabilistic-statistical simulation of the non-linear mechanical behavior of a woven SiC/SiC composite. *Composites Science and Technology*, 56(803–808), 1996.
- [2] Lissart N & Lamon J. Damage and failure in ceramic matrix minicomposites: experimental study and model. *Acta mater.*, 45(1025-1044), 1997.
- [3] Bolzon G. Formulation of a triangular finite element with an embedded interface via isoparametric mapping. *Computational Mechanics*, 198(3670-3681), 2001.
- [4] Liu W, Yang Q.D, Mohammadizadeh S. & Su X.Y. An efficient augmented finite element method for arbitrary cracking and crack interaction in solids. *Int. J. Numer. Meth. Engng*, 99(438–468), 2014.
- [5] Liu X, Duddu R & Waisman H. Discrete damage zone model for fracture initiation and propagation. *Engineering Fracture Mechanics*, 92(1–18), 2012.

## On the treatment of crack boundary conditions in phase field models of dynamic fracture

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Computation of complex crack patterns is a challenging task in fracture mechanics and the phase field approach thereto shows excellent capabilities. A scalar phase field variable *s* degrades the elastic energy density in a regularized representation of cracks in isotropic materials. Different methods are used so far to model the property of a crack to weaken a material only in tension and shear. One approach is the decomposition of strain in negative spherical, positive spherical and deviatoric parts, in which only the energy density of the latter two is degraded by the phase field [1]. Another method is based on the spectral decomposition of strain [2] and the split into positive and negative principal strains. However, both models avoid to take an explicit representation of the crack direction into account.

This contribution presents a new approach which takes the direction of the crack into account. For an exact representation of cracks, the stress component normal to the crack surface should only be non-zero in case of compression and shear stresses acting on a frictionless crack surface should vanish. In order to satisfy these conditions stresses and strains are represented with respect to a coordinate system oriented with the crack. The crack normal direction is computed from the gradient of the phase field, which is already available from the description of the smeared surface energy. Moreover, the degrading function of the phase field acts only on some parts of the elastic energy density. The derivation of the stress tensor leads to an elasticity tensor displaying transverse isotropy with symmetry about the crack normal. The new model is able to treat crack opening and closure in a correct manner. The performance of the proposed model is demonstrated by means of numerical simulations, including representative tests for stationary cracks as well as simulations for dynamically propagating cracks.

- [1] H. Amor, J.-J. Marigo, C. Maurini. Regularized formulation of the variational brittle fracture with unilateral contact: Numerical experiments. *Journal of the Mechanics and Physics of Solids*, 57(8), 1209–1229, 2009.
- [2] C. Miehe, F. Welschinger, M. Hofacker. Thermodynamically consistent phase-field models of fracture: Variational principles and multi-field FE implementations. *Int. J. Numer. Meth. Engng.*, 83(10), 1273–1311, 2010.

## An adaptive body-fitted monolithic method for modeling the fracture of heterogeneous microstructures

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In a Finite Element (FE) framework, multiple paths are available to model heterogeneous materials in their specific morphologies and behaviors. Multi-body approaches, which are more straightforward and consist in meshing each phase of the material independently, raise the important problem of handling contact when the interfaces are numerous and have a complex morphology. Monolithic approaches avoid this issue by admitting continuous displacement fields at interfaces and discretizing the whole material, all heterogeneities included, in the same FE mesh. For discontinuous displacement fields in a monolithic approach, enriched FE methods such as the discontinuous Galerkin method or the eXtended FE Method have been developed [1].

In a recent work [3, 4], a new body-fitted monolithic approach was proposed and applied to the study of void coalescence at the microscale, which plays a key role in ductile fracture. To reduce the complexity and the cost of remeshing operations, numerous monolithic methods are based either on enriched FE methods or on an Eulerian framework with an implicit representation of interfaces [2]. The new Lagrangian methodology developed in [3] focused on improving remeshing operations to enable the meshing and remeshing of explicit interfaces. To model discontinuities, the single mesh used in these developments also embodied the void phase that appears at the onset of fracture, which was then modeled by an appropriate fluid behavior, as in [5, 6].

The study of ductile fracture at the microscale is an interesting topic to test the robustness and validity of this method. This application involves existing interfaces between the different phases (matrix, inclusions and voids) which evolve due to deformation, and also new interfaces that appear during the simulation due to inclusion fracture, interface debonding between matrix and inclusions, and void coalescence. Regarding void coalescence in plane strain, promising results were proposed in [3], and proved the ability of the method to handle complex void arrangements, especially during the apparition and propagation of coalescence. This framework was then completed with the modeling of void nucleation and applied to 3D microstructures.

- [1] N. Moës, J. Dolbow, T. Belytschko. A finite element method for crack growth without remeshing. *International Journal for Numerical Methods in Engineering*, 46, 131-150, 1999.
- [2] M. Sussman, P. Smereka, S. Osher. A Level Set Approach for Computing Solutions to Incompressible Two-Phase Flow. *Journal of Computational Physics*, 114, 146-159, 1994.
- [3] M. Shakoor, M. Bernacki, P.-O. Bouchard. A new body-fitted immersed volume method for the modeling of ductile fracture at the microscale: analysis of void clusters and stress state effects on coalescence. *Engineering Fracture Mechanics*, Submitted, 2015.
- [4] M. Shakoor, M. Bernacki, P.-O. Bouchard. Micromechanical modeling of ductile fracture mechanisms using a new body-fitted immersed volume method. *International Conference on Computational Modeling of Fracture and Failure of Materials and Structures (CFRAC 2015)*, 3-5 June, 2015, Paris, France.
- [5] E. Roux, M. Bernacki, P.-O. Bouchard. A level-set and anisotropic adaptive remeshing strategy for the modeling of void growth under large plastic strain. *Computational Materials Science*, 68, 32-46, 2013.
- [6] E. Roux, M. Shakoor, M. Bernacki, P.-O. Bouchard. A new finite element approach for modelling ductile damage void nucleation and growth analysis of loading path effect on damage mechanisms. *Modelling and Simulation in Materials Science and Engineering*, 22, 075001, 2014.

## A new generalized finite element method for two-scale simulations of propagating cohesive fractures in 3-D

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We present a novel generalized finite element method with global-local enrichments (GFEM<sup>gl</sup>) for two-scale simulations of propagating cohesive fractures in three dimensions. A non-linear cohesive law is adopted to capture objectively the dissipated energy during the process of material degradation without the need of adaptive remeshing at the macro scale or artificial regularization parameters. The cohesive crack is capable of propagating through the interior of finite elements in virtue of the partition of unity concept provided by the generalized/extended finite element method, and thus eliminating the need of interfacial surface elements to represent the geometry of discontinuities and the requirement of finite element meshes fitting the cohesive crack surface. The proposed method employs fine-scale solutions of non-linear local boundary-value problems extracted from the original global problem in order to not only construct scale-bridging enrichment functions but also to identify damaged states in the global problem, thus enabling accurate global solutions on *coarse* meshes. This is in contrast with available GFEM<sup>gl</sup> in which the local solution field contributes only to the kinematic description of global solutions. The robustness, efficiency, and accuracy of this approach are demonstrated through representative numerical examples.



Three-point bending test solved with GFEM<sup>gl</sup> for cohesive fractures.

## A stabilization technique for nearly singular extended finite elements used for static and dynamic crack analysis

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The eXtended Finite Element Method (XFEM) is a widely used technique for the simulation of cracks, heterogeneities and other problems in solid and fluid mechanics showing discontinuities in the field variables or their derivatives or localization phenomena of any type. It has shown to be very efficient and attractive for two and three dimensional simulations. However, the geometry of the discontinuity in conjunction with the mesh geometry and the applied XFEM enrichment scheme might in certain cases lead to very badly conditioned or even singular coefficient matrices, in general due to (near) linear dependencies between standard and enriched degrees of freedom. This can easily be avoided by slightly changing the crack geometry [1]. However, even this slight change of the geometry in general is undesired. During the last years several stabilization techniques have been proposed in literature based on modifications of the enrichment functions [2], special preconditioning techniques [3], [4], or direct modifications of the resulting equation system [5].

In this contribution advantages and disadvantages of some of the stabilization techniques are presented. Additionally a simple but very efficient regularization method presented in [6] is combined with other stabilization techniques and extended to dynamic crack simulation problems. The technique is based on an eigenvalue decomposition of the element stiffness matrices of selected fully enriched elements and a stabilization of the subspace that causes ill-conditioning. This regularization technique has very little effect on the solution of the resulting equation system but significantly improves the condition number and is thus ideal especially for large three dimensional eXtended Finite Element problems. This is demonstrated with a number of numerical examples for static and dynamic crack simulations.

#### References

 D.Müller-Hoeppe. A multiscale method for fracturing solids. (Dissertation) ISBN 978-3-941302-05-1 (2012)

- [2] I. Babuška and U. Banerjee. Stable generalized finite element method (SGFEM). *Computer Methods in Applied Mechanics and Engineering* 201-204, 91-111, 2012.
- [3] E. Béchet, H. Minnebo, N. Moës and B. Burgardt. Improved implementation and robustness study of the X-FEM for stress analysis around cracks. *International Journal for Numerical Methods in Engineering*, 64, 1033-1056, 2005.
- [4] A. Menk and S. Bordas. A robust preconditioning technique for the extended finite element method. *International Journal for Numerical Methods in Engineering*, 85, 1609-1632, 2011.
- [5] T. Strouboulis, I. Babuška and K. Copps. The design and analysis of the generalized finite element method. *Computer Methods in Applied Mechanics and Engineering*, 181, 43-69, 2000.
- [6] S. Loehnert. A stabilization technique for the regularization of nearly singular extended finite elements. *Computational Mechanics*, 54, 523-533, 2014.

# Cohesive crack with friction growing at the interface between dissimilar isotropic materials

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The behaviour of many concrete structures, such as concrete dams or underground repositories for nuclear waste storage, strongly depends on a cohesive crack with friction growing at the interface with the rock foundation. In the quasi-static analysis of these problems traditional iterative (Newton Raphson) methods fail to converge to the equilibrium condition, even by using the so called path-following arc-length method [1].

In order to overcome these difficulties, according to a method called '*fictitious crack length control scheme*', for a predefined position of the Fictitious Crack Tip(shortened FCT), it is useful to enforce two [2] or more terms of the asymptotic expansion [3], [4], [5] along the Fracture Process Zone.

In this way it is possible to stabilize the above mentioned numerical process in order to obtain the external load multiplier which is associated with the predefined position of the FCT. Therefore no previous converged step is required. From this point of view this method is similar to the non-incremental LArge Time INcrement (shortened LATIN) methods [6], [7].

With this approach it is possible to analyse cracking processes occurring at a macro structural level on the basis of mechanical properties measured in the laboratory and having a specific microstructural meaning.

In the present paper some new results related to a gravity dam model proposed as a benchmark by the International Committee on Large Dams [8] are presented.

- [1] E. Riks, C.C. Rankin, and F.A. Brogan: On the solution of mode jumping phenomena in thin-walled shell structures. *Computer Methods in Applied Mechanics and Engineering*, 136(1):59–92, 1996.
- [2] A. Carpinteri and S. Valente and G. Ferrara and G. Melchiorri: Is mode II fracture energy a real material property? *Computers and Structures*, 14(3):397–413, 1993.
- [3] B.L. Karihaloo and Q.Z. Xiao: Asymptotic fields at the tip of a cohesive crack. *International Journal of Fracture*, 150:55–74, 2008.

- [4] F. Barpi and S. Valente: The cohesive frictional crack model applied to the analysis of the dam-foundation joint. *Engineering Fracture Mechanics*, 77:2182–2191, 2010.
- [5] A. Alberto and S. Valente: Asymptotic fields at the tip of a cohesive frictional crack growing at the bi-material interface betweena dam and the foundation rock. *Engineering Fracture Mechanics*, 108:135–144, 2013.
- [6] Ph. Boisse, P. Ladeveze, and P. Rouge: A large time increment method for elastoplastic problems. *European Journal of Mechanics A/Solids*, 8(4):257–275, 1989.
- [7] B. Vandoren, K. De Proft, A. Simone, and L. J. Sluys: A novel constrained LArge Time INcrement method for modelling quasi-brittle failure. *Computer Methods in Applied Mechanics and Engineering*, 265:148–162, 2013.
- [8] ICOLD: Imminent failure flood for a concrete gravity dam. In 5th International Benchmark Workshop on Numerical Analysis of Dams, Denver, CO, 1999.

### Modelling of Damage in Reinforced Beam-Column Elements with Embedded Discontinuities

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A beam-column element with embedded discontinuities is formulated and implemented for modelling the formation of hinges, which includes the rotational, transversal displacement and axial displacement jumps. The variational formulation includes the strain energy of a beam [1] and a bar, both elements with embedded discontinuities. This element uses constitutive models to independently include the moment, shear force and axial force. These constitutive models are formulated based on damage mechanics and experimental test reported in the literature. The frame element with embedded discontinuities and the constitutive models where implemented in the Finite Element Analysis Program (FEAP) developed by [2].

Some numerical examples are presented to show the capability of the beam-column element with embedded discontinuities to release energy by the development of hinges. These examples include tunnels, beams and frames under loads, which induces damage in the elements. Numerical modelling of segmented tunnels are discretized with beamcolumn elements, which are supported on spring elements with the ground properties. Spring elements were also used for modelling the joints between the segments in the circumferential direction and the ring joints which connect parallel adjacent rings. The numerical load vs displacement curves are congruent with numerical and experimental results reported in the literature.

- [1] G. Juárez-Luna and G.A. Ayala. Finite element variational formulation for beams with discontinuities. Finite Elements in Analysis and Design, 54, 37-47, 2012.
- [2] L.R. Taylor. A finite element analysis program (FEAP) v8.2, Department of Civil and Environmental Engineering, University of California at Berkeley, Berkeley, CA 2008.

#### Mixed-mode fracture in strong discontinuities

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Damage is usually considered as a deformation driven process. However, fracture criteria, such as crack initiation and crack evolution, are typically defined in the stress or traction space. As a consequence, in most cases a failure surface is not correctly predicted by the models.

In this contribution, different damage models are studied for mixed-mode fracture in the context of strong discontinuities. First, similar to the work presented in [1], deformationbased models are adopted. The corresponding predicted limit surface in the traction space is found to be linear, with an inclination dependent on the elastic stiffness coefficients, which is a limitation. An energy-based model is considered next. Both isotropic damage evolution and non-isotropic damage evolution are introduced. The isotropic version gives rise to a symmetric constitutive relation. However, in this case, circular limit surfaces are predicted in the traction space. A more refined version is proposed, in which the internal damage variable is also made dependent on the traction field: a better approximation of the limit surface is obtained; however, symmetry of the tangent constitutive tensor is lost. Thirdly, a traction-based model is proposed. In this case the limit surface in the traction space is enforced. Different damage evolution laws are analysed to cope with both mixed-mode fracture and shear bands under compression.

Finally, dilatancy is taken into account. Examples are shown for the three concepts, with and without dilatancy, and are compared to experimental data.

#### References

[1] Alfaiate, J. and Wells, G. N. and Sluys, L. J. On the use of embedded discontinuity elements with crack path continuity for mode-I and mixed-mode fracture *Engineering Fracture Mechanics* 69(6), 661-686, 2002.

## Determination of the fracture parameters in three-dimensional structure by the finite element method

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Abstract

A lot of structures of Civil Engineering and construction Engineering are cracked more or less. The analysis of the evolution of their behavior and the possible risk of irreversible disorders (fracture...) is a function of the existing cracks and their potentiality to evolve. Several researchers studied this phenomenon and developed fracture mechanics theory, permitting to analyze the evolution of cracks until the possible ruin of the structure.

In the present research, we used the equivalent domain integral (EDI) to evaluate the stress intensity factor along the crack front using the finite element method. Based on the equivalent domain integral method with auxiliary fields, an interaction integral is also derived to extract the second fracture parameter is called the T–stress for a threedimensional isotropic cracked solid.

A program in Ansys 11.0 (in batch mode) is elaborated to modeling and to analyzing the three-dimensional isotropic cracked structure. A second program is also elaborated in Fortran 90 called ECAPE2. This program permits to determine the distributions of stress intensity factor and *T*-stress components, along a treedimensional crack front.

We validated these programs while treating two models the more frequently met in fracture mechanics that are respectively a set of plates center-cracked and other set of plates to double edge crack under pure mode-I loading conditions.

- [1] K. -J. Bathe, *Finite Element Procedures in Engineering Analysis*, Prentice-Hall Inc., Englewood Cliffs, NJ, U.S.A., 1982.
- [2] G. P. Nikishkov and S. N. Atluri, An equivalent domain integral method for computing crack-tip integral parameters in non-elastic, thermo-mechanical fracture. *Engineering Fracture Mechanics*, 26, No.6, pp.851-867, 1987.
- [3] T. L. Anderson, *Fracture Mechanics: Fundamentals and Applications*, Second Edition. CRC Press, Boca Raton, Florida, U.S.A., 1995.
- [4] K. -J. Bathe, *Finite Element Procedures in Engineering Analysis*, Prentice-Hall Inc., Englewood Cliffs, NJ, U.S.A., 1982.
- [5] D.François, A.Pineau & A.Zaoui : Comportement mécanique des matériaux (viscoplasticité, endommagement, mécanique de la rupture, mécanique du contact). Edition Hermes 1995.

- chapter 2 -

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## MS2: Advances in the Experiment-Modeling Dialog



## X-DIC for ductile tearing monitoring using ultra high resolution cameras

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Ductile tearing simulation is still a challenge when you want to follow the process of rupture from the initiation of damage until the final macroscopic crack. Even if the modeling and the simulations are becoming increasingly precise, experimental tests should also allow to discriminate between different models as accurately. The main difficulty being to succeed of measuring strains along deacreasing scales. When using Digital Image Correlation (D.I.C.) [1], the smaller subset size is about  $20 \times 20$  pixels to measure strains accurately. If one wants a necked zone covered per 10 subsets, you need 200 pixels in the bandwith and again almost 10 times more pixels for the whole specimen (about 2000 px of width). Then, 4MPx camera is the lower limit of resolution in that case [2]. But, if one expects to track the crack localization (in mode I or II) inside a necked zone, you will need 10 times more pixels in the bandwidth. In the present work, we show that it is possible to doing this, using ultra high resolution cameras (from 29 Mpx images where  $1px = 1\mu m$ to 260 Mpx when the pixel shifting technic is used). A new specific test called *flounder* eyes test will be presented. A specimen perforated of two asymmetric and different radius holes is subjected to vertical tension. The very slow loading allow shooting the specimen all along the onset of necking and crack propagation in mode I outer of the holes and in mode II between the holes (see fig. 1). The description and results of these tests have been posted in digital format in an open web data base [3] and may be used as a benchmark for ductile tearing. An example of displacement field is given in fig. 2.



FIGURE 1 – Three speckle images of a flounder eyes test on S235 steel during tearing.





#### Références

- [1] Réthoré, J., Hild, F., Roux, S. International journal for numerical methods in engineering. *IJNME*, 73(2), 248-272, 2008.
- [2] Wu, T., Coret, M., Combescure, A. Strain Localisation and Damage Measurement by Full 3D Digital Image Correlation.*Strain*, 47(1), 49-61. 2011.
- [3] http://radome.ec-nantes.fr/, Janvier 2015

## Modelling of concrete fracture at aggregate level using FEM and DEM

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Fracture is a fundamental phenomenon in quasi-brittle and brittle materials. It is a major reason of mechanical damage under loading that contributes to a significant degradation (reduction) of the material strength. It is highly complex due to a heterogeneous structure of brittle materials over many different length scales, changing in e.g. concrete from a few nanometres (hydrated cement) to the millimetres (aggregate particles). Therefore, the material heterogeneity should be taken into account when realistically modelling the material behaviour. An understanding of a fracture process is of major importance to ensure the safety of the structure and to optimize the behaviour of material.

The paper describes numerical 2D results of fracture at the aggregate level in notched concrete beams under quasi-static three-point bending. Two different numerical approaches were used: a continuum [1] and discrete one [2]. Within continuum mechanics, the simulations at the aggregate scale were carried out with the FEM based on a damage constitutive model enhanced by a characteristic length of micro-structure by means of a non-local theory [1]. The model has 6 material constants. As a discrete approach, the three-dimensional spherical discrete element model YADE was used [2]. The model takes advantage of the so-called soft-particle approach (i.e. the model allows for particle deformation which is modelled as an overlap of particles). The model has 5 material constants.

Concrete was considered as a four-phase body. It included aggregate particles, cement matrix particles, interfacial transitional zones (ITZs) and voids. The concrete microstructure was directly taken from the real specimens based on 3D images (aggregate, cement matrix and voids) with the aid of the x-ray micro-tomography and 2D images (ITZs) from the micro-scope with a very high resolution (Fig.1). In particular, the presence of aggregate particles and ITZs is important since the volume fraction of aggregate can be as high as 70-75% and the ITZs with the thickness of about 50 µm are always the weakest region in usual concretes (wherein cracking starts because of their higher porosity). Initially, a comprehensive calibration process was performed based on uniaxial tensile tests. A satisfactory agreement between the experiments and numerical calculations with respect to the force-deflection diagram and propagation way of the crack above the notch was obtained (Fig.2). The crack moved mainly by ITZs (sometimes it moved however through a weak aggregate particle). In addition, some micro-structural events (force chains, local void ratio change, aggregate rotation and vortex-structures) were studied in detail by DEM [2].



**Figure 1**: Concrete micro-structure: a) 3D image of concrete micro-structure by x-ray micro-tomography and b) 2D image of ITZ by scanning electron microscope



**Figure 2**: Comparison between experiments (a) and numerical approaches (b, c): a) 2D view on cracked concrete surface from x-ray micro-tomography, b) localized zone by FEM based on non-local strain and c) crack by DEM based on broken contacts

- L. Skarżyński and J. Tejchman, Calculations of fracture process zones on mesoscale in notched concrete beams subjected to three-point bending. European Journal of Mechanics/A Solids 29, 746-760, 2010.
- [2] M. Nitka and J. Tejchman. Modelling concrete behaviour in uniaxial compression and tension with DEM. Granular Matter, doi: 10.1007/s10035-015-0546-4, 2014.

## Heterogeneity informed quantitative micromechanical approach of ductile fracture in 6xxx aluminium alloys.

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Ductile fracture results from the nucleation, growth and coalescence of small internal cavities. In aluminium alloys, the void population generally nucleates by the fracture of iron rich intermetallic particles. The objective of this study is to understand and model the effect of microstructure heterogeneities on damage accumulation in three 6xxx series aluminium alloys. The three alloys, i.e. Al 6005A, Al 6061 and Al 6056, exhibit a volume fraction of iron rich particles close to 1%. However, samples of similar yield strengths, owing to appropriate heat treatments, show major differences in the true fracture strain for these three alloys.

A cellular automaton model, involving a high number of particles with distribution of position, sizes and void nucleation stress is developed to predict fracture. The model treats local interaction between neighbouring cavities in a simplified way and captures cluster effects on coalescence. The model parameters are extracted from a detailed microstructure analysis. High resolution 3D X-ray synchrotron tomography is used to characterize the size and position distribution of the iron-rich intermetallics and initial cavities in the three alloys. In addition, a statistical study performed on polished fractured tensile samples allows extracting nucleation stresses and the probability of fracture as a function of the size of the intermetallic particle. The model quantitatively predicts all the fracture strain using a single void nucleation stress distribution as a function of particle size with no other fitting parameters. This shows that the key element setting the fracture strain is the effect of particle size distribution and spatial distribution on void nucleation and coalescence.

# Mecanical behavior and failure of irradiated filled elastomer using molecular dynamics

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The failure of elastomer is described here in a context of studying the ageing of irradiated materials. The failure depends on the microscopic architecture of the material (cross-link density, free-ended chains, local defects...). Molecular dynamics simulations are used to control accurately this architecture. Elastomers are created using the polymer classical coarse-grain model [1], radical-like polymerisation [2] and random cross-linking with to-pological constrains. Tensile tests are then simulated up to their rupture. In a second time, irradiation ageing effects are added in the simulation. The tests results show a significant effect of the aschitecture variations on the elastic behavior and the failure.

#### Références

- [1] Kremer, K.; Grest, G. S. Dynamics of entangled linear polymer melts : A moleculardynamics simulation *The Journal of Chemical Physics*, 1990, 92, 5057-5086
- [2] Gao, Y.; Liu, J.; Shen, J.; Zhang, L.; Cao, D. Molecular dynamics simulation of dispersion and aggregation kinetics of nanorods in polymer nanocomposites *Polymer*, 2014, 55, 1273-1281

## Extraction of SIFs and crack tip detection for curved cracks using digital images

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Extracting stress intensity factors (SIFs) and detecting the position of the crack tip using digital image correlation (DIC) have been the topic of numerous papers. Most of the existing approaches consider a projection of a measured displacement measured onto a basis allowing for estimate SIFs (see e.g. [1]. The position of the crack tip is either considered as a minimization parameter of the projection or directly estimated from the first super-singular field amplitude of the Williams' series as proposed in [2]. Based on the latter framework, it is shown in [3] that the SIFs best estimate is obtained when SIFs are among the unkwon of the DIC algorithm. The major limitation of all these techniques concerns curved cracks. Indeed, for the Williams' series to be used, the size of the extraction domain must be small compared to the crack curvature. However, the ill-posed nature of the extraction problem makes the uncertainty on SIFs and crack tip position increase when the extraction domain size decreases. There is thus a need for an additional effort to deal with curved cracks with low uncertainty levels. The proposed approach is based on a finite element DIC formulation with elastic regularization. Further, an adaptive meshing strategy is used to adapt the mesh to the crack morphology and to properly mesh a circular domain at the crack tip. Within this inner domain, the finite element degrees of freedom are condensed onto the Williams' series coefficients. The elastic regularization adopted in the outer domain, allows to decrease the size of the extraction domain significantly while a low uncertainty level is maintened. The performance of the method is evaluated using an artifical test and its ability to track crack propagation in mixed mode is illustrated with a real experiment on a PMMA plate.

- [1] McNeill S, Peters W, Sutton M. Estimation of stress intensity factor by digital image correlation. *Engineering Fracture Mechanics* 1987; **28**(1):101–112.
- [2] Hammam R, Hild F, Roux S. Stress intensity factor gauging by digital image correlation: Application in cyclic fatigue. *Strain* 2006; **43**(3):181–192.
- [3] Roux S, Réthoré J, Hild F. Digital image correlation and fracture: an advanced technique for estimating stress intensity factors of 2D and 3D cracks *Journal of Physics* D: Applied Physics 2009; 42.

## Numerical modeling of propagating compaction bands in brittle granular media

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During the uniaxial tension or compression of a material like foam, bone or metal, plastic deformation in the specimen can localize in bands. In some alloys, these bands can move through the sample leading to a serrated stress-strain curve known as jerky flow or Portevin–Le Châtelier effect [1]. Recently, the same kind of phenomenon of propagating compaction bands has been observed in the compression of brittle granular materials, like snow [2] or puffed rice [3].

To better understand the apparition of propagating compaction bands in brittle granular materials, we developed a discrete 2-dimensional numerical simulation. A lattice of springs is compressed at constant velocity. Each spring has a maximum force it can support, beyond which it breaks, i.e. the force it carries drops to zero and its stiffness and strength are increased.

This simple configuration leads to a wide variety of velocity patterns in the specimen. If at low velocity the deformation of the sample is generally erratic, at medium velocity a propagating deformation band can form. At larger velocity the medium deform almost homogeneously. Depending of the parameters, other behavior have been observed, like the propagation of multiple bands. These simulations are confronted to experimental observations on puffed rice showing very similar patterns, related to the location of breakage in the sample. The selection of the observed deformation pattern is studied in relation with the characteristic times in the specimen, which could lead to a better understanding of the condition of apparition of propagating compaction bands in brittle materials.

- A. H. Cottrell, LXXXVI. A note on the Portevin-Le Chatelier effect. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 44(355), 829–832, 1953.
- [2] T.W. Barraclough *et al.* Propagating compaction bands in confined compression of snow: Experiment and Modelling. *arXiv preprint arXiv:1501.02184*.
[3] J. R. Valdes, F. L. Fernandes, I. Einav. Periodic propagation of localized compaction in a brittle granular material. *Granular Matter*, 14(1), 71–76, 2011.

# Experimental database for mixed-mode crack propagation in concrete: comparison between experimental and numerical results using fullfield measurements

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In order to experimentally validate concrete damage and fracture models, identify its parameters and better characterize the concrete behaviour during mixed-mode crack propagation, multiaxial tests are developed. Information considering mixed-mode crack propagation in concrete is vital to the development of the damage and fracture models, however this kind of tests are seldom found in literature. Among these tests, the results obtained by Nooru-Mohamed (1992) [1] and Winkler (2001) [2] are often used to validate models since nontrivial crack paths are obtained under mixed-mode loading conditions. However, the comparison is based just on the final crack pattern and the boundary conditions that are prescribed in the numerical simulations are the theoretical ones. The present goal is to perform rich and discriminating tests by using state of the art techniques, where the experimental boundary conditions are directly measured during crack propagation. The loadings are applied using a hexapod testing machine controlled by a 3D displacement system [3] and the cracking state is analysed via digital image correlation.

With the proposed experimental setup several loading histories are analysed: proportional multiaxial loading histories and non-proportional ones. In the present case, the experiments developed are mainly used for validation purposes. They can also be used to identify some of the damage models parameters that are not accessible through standard tests (e.g., equibiaxial tensile strength). The experimental results are confronted with numerical simulations performed with a nonlocal (i.e., gradient) damage model [4], [5]. The present work underlines the importance of using accurate boundary conditions, estimated from full field measurements [6], to perform numerical simulations that reproduce the experimental results.

It is believed that these experiments are not only challenging the chosen damage model but more generally a wide class of damage and fracture models. The fact that the tests have been monitored very extensively should allow for a thorough investigation in which kinematic fields and static data can be directly compared with numerical simulations.

- [1] M.B. Nooru-Mohamed. "Mixed-mode fracture of concrete: an experimental approach". PhD thesis, Technische Universteit Delft, 1992.
- [2] B. Winkler. "Traglastuntersuchungen von unbewehrtenund bewehrten Betonstrukturen". PhD thesis, University of Innsbruck, 2001.
- [3] J. Le Flohic, V. Parpoil, S. Bouissou, M. Poncelet, H. Leclerc. A 3D Displacement Control by Digital Image Correlation for the Multiaxial Testing of Materials with a Stewart Platform. Exp. Mechanics, 54(5) 817-828, DOI 10.1007/s11340-013-9837z, 2014.
- [4] E. Lorentz, S. Andrieux. A variational formulation for nonlocal damage models. Int. J. Plast. 15, pp. 119-138, 1999.
- [5] E. Lorentz, V. Godard. Gradient damage models: Toward full-scale computations. Comput. Meth. Appl. Mech. Eng. 200, pp. 1927-1944, 2011.
- [6] M.A. Sutton. Computer Vision-Based, Noncontacting Deformation Measurements in Mechanics: A Generational Transformation, Appl. Mech. Rev., 65(5), 23pp, 2013.

# Mixed mode fracture of mortar joints in masonry buildings

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Masonry is an anisotropic and heterogeneous material, in which mortar joints act as planes of weakness. Many variables influence the mechanical behaviour of masonry, as material properties of bricks and mortar, geometry of bricks, dimensions of joints, brick arrangements etc... It is consequently quite a challenge to accurately represent the behaviour of each block and cracking of the joints.

Discrete Element Method (DEM) allows to model collections of rigid or deformable bodies in interactions, and thus seems appropriate to model such a discontinuous material as masonry. The LMGC90 [1] is a discrete element software, developped by Montpellier University, and based on the "Non Smooth Contact Dynamics" method. It allows to deduce a global behaviour from the behaviour of each object, taking into account the interactions between each body. An accurate description of interactions is thus essential to well describe the global behaviour of the structure. To model masonry, we chose to average the mortar joints and brick/mortar interfaces to one cohesive interface.

The failure of the interface is assumed to be quasi-brittle, described by a cohesive zone model with an elastic part, followed by a softening branch in which stress decreases, damage propagates, and fracture energy is consumed. To approximate the softening part, we adopt the bilinear model proposed by Morel et. al. [2]. The global fracture energy  $J^c$  is divided into two fracture energies, that represent mobilized mechanisms : microcracking  $J^{\mu}$  and crack bridging  $J^b$ . The shape of cohesive function is then given by :

- (*i*) the cohesive fracture energy  $J^c$
- (*ii*) the energy distribution between the two cohesive energies  $J^{\mu}$  and  $J^{b}$ , defined from the ratio  $\Phi = J^{\mu}/J^{b}$
- (*iii*) the critical opening  $\delta^{u}$
- (*iv*) the maximal stress  $\sigma^c$
- (v) the initial stiffness  $K^0$

To better represent real mechanisms, which are combinations of mode I (traction) and mode II (shear), we formulate the above cohesive function with mixed mode. The mixed mode cohesive model is deduced from the parameters given above ((*i*) to (v)) of each pure modes. Using the work of De Moura [3], we use a quadratic stress criterion to determine

damage initiation (opening at maximal stress), and a fracture energetic criterion to manage crack growth and damage propagation.

To validate the proposed cohesive zone model, an experimental campaign has been led, with masonry made of limestone ashlar and lime mortar joints, typical of old buildings around the city of Bordeaux. First, traction tests on one mortar joint between two blocks, and triplets shear tests were performed to obtain values corresponding to pure modes parameters. A half-scaled masonry wall has then been tested under quasi-static shear loading. A discrete element model of the tested wall was realized. Experimental datas from traction and shear tests allowed to define the cohesive interface law. The damage propagation in the simulation shows good agreement with test cracking. We also obtain similarities between experimental and numerical global behaviours.

# Références

- [1] F. Dubois, M. Jean. The non smooth contact dynamic method : recent LMGC90 software developments and application. *Analysis and Simulation of Contact Problems*, 375-378, 2006.
- [2] S. Morel, C. Lespine, J-L. Coureau, J. Planas, N. Dourado. Bilinear softening parameters and equivalent LEFM R-curve in quasi-brittle failure. *International Journal of Solids and Structures*, 47, 837-850, 2010.
- [3] M.F.S.F. de Moura, R.D.S.G. Campilho, J.P.M. Gonçalves. Mixed-Mode Cohesive Model Applied to the Simulation of the Mechanical Behaviour of Laminated Composite Adhesive Joints. *Journal of Adhesive Science and Technology*, 23 (10-11), 1477-1491.

# Integration of Micromechanical Measurements to Support Microstructure-Based Modeling

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In this work we combined two different experimental techniques, x-ray computed tomography (CT) and quantitative acoustic emission (AE) in order to extract robust estimates of micro mechanical properties at different stages of loading and damage. The techniques were applied to the specific problem of energy dissipation in steel fiber reinforced ultra high performance concrete (UHPC) subjected to quasi-static loading. In previous work[1], analysis of CT images of damaged concrete allowed us to deduce the contribution of different toughening mechanisms in a UHPC reinforced with 30 mm hooked steel fibers. Specifically, results showed that beam specimens subjected to 3-point bending, matrix cracking actually dissipated more energy than fiber pullout. However, later work showed, not surprisingly that this depends on the extent of specimen loading.

In order to identify different energy dissipation mechanisms as a function of damage level, AE activity was monitored during the test. The goal was to deduce different types of damage through classification of the AE waveforms. As a starting point for classification, AE activity was monitored during simple tests of unreinforced specimens subjected to 3 point bending, and dogbone tension specimens subjected individual fiber pullout. The former was used to establish AE characteristics of matrix cracking, while the latter was used to establish AE characteristics of fiber pullout. A wavelet decomposition was used to characterize signals from the simple tests. These characteristics were then used to identify signals that corresponded to the matrix cracking as compared to the pullout in the fiber reinforced specimens. Results showed that, not surprisingly, early stages of damage are dominated by matrix cracking, while fiber pullout energy dissipation increases as the specimen is further deformed. By combining the two techniques, we can produce a fairly clear quantitative picture of energy dissipation. Such information can be part of a robust model validation program.

# Références

 K. Trainor, B. Foust, and E. Landis Measurement of Energy Dissipation Mechanisms in Fracture of Fiber-Reinforced Ultrahigh-Strength Cement-Based Composites. *Journal of Engineering Mechanics*, 139(7), 771-779, 2013.

# Crack propagation in inelastic pre-strained specimen

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Fundamental knowledge of crack propagation is essential for safety assessments of aero engine critical parts. The total life of a compressor or turbine disc can be split into the crack initiation phase and the subsequent crack propagation phase.

Highly stressed features especially those ones with a high surface stress concentration factor tend to be inelastically stressed in the first flights. After the initial redistribution of stress the material is cycling along the Hooke's line in the subsequent flights. This phenomenon is known as elastic shake-down.



**Fig.1**:Elastic shake-down condition in a stress concentration feature after cyclic loading (here: cruciform specimen) [1]

Since linear elastic fracture mechanics is only valid in elastic stress regimes it has been investigated whether it's usage can be extended to elastic shake down conditions.

The work presented here focuses on crack propagation in inelastic pre-strained specimens. A test logic has been developed to identify the effect of the inelastic pre-straining, which could change the material behaviour via hardening or softening of the material. In a second step the effect of inhomogeneous pre-straining is investigated. This case is considered to be relevant for component features that exhibit elastic shake down.

The experimental test program comprised specimen tests using the corner crack specimen design, which due to its similarity to real cracks is the standard in aero engine crack propagation testing. Furthermore, a modified notched corner crack specimen was tested, which has been developed for the inhomogeneous pre-strained features.

Two typical engine disc materials were used in the test programme, Udimet 720 Li and Titanium 6246. The results, which include stress intensity factor solutions for both specimen types, indicate that the inelastic pre-straining has no significant effect on the propagation of the specimen and therefore the methods of linear elastic fracture mechanics can be applied to features exposed to elastic shake down conditions.

# References

[1] Schlums, H.; Rothkegel, W.; Thomas, A.andVarga, Á.: Evaluation of Lifetime Prediction Methods for Multiaxial Fatigue on the Basis of Biaxial Testing of the Disc Alloy Udimet720Li at elevated Temperatures, FATIGUE 2007 – 6th Int. Conf. on Durability and Fatigue, Cambridge, UK, 26-28 March 2007

# Mode I-III Decomposition of the *J*-integral from Digital Image Correlation Displacement Data

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A large number of failures in engineering structures occur under complex loading leading to brittle fracture (e.g. pressure vessels, pipe lines, aerospace). A promising technique to quantify fracture is the evaluation of the *J*-integral on full-field displacement data obtained by digital image correlation (DIC) [1]. However, the evaluation of brittle fracture frequently requires separation of the *J* equivalent stress intensity factor (SIF) into separate mode I, II and III components. As these are not readily available from standard *J*-integral evaluations, specialised methods such as the interaction integral [2] or field fitting approaches [3] are usually required. The decomposition method, proposed by Ishikawa et al [4], enables direct computation of mode I, II and III *J*-integrals, which may be converted to SIFs in linear elastic materials.

This study investigates the applicability of the decomposition method to the extraction of mode I-III SIFs from DIC displacement fields. The method is tested and validated on mixed mode I-III Williams series displacement fields. SIF estimation errors were determined by comparing SIFs from analytical fields with those obtained by the *J*-integral decomposition method. The accuracy of the method was investigated with respect to three types of synthetically generated DIC displacement error:

- Random displacement noise
- Rigid body misalignment: translational and rotational
- Failed correlation (missing displacement data) in crack regions

It was found that, within the error margins of most DIC algorithms, displacement noise and rigid body misalignments have negligible effect on SIF estimation accuracy (less than 1 %). Significantly more prominent errors occurred as a result of missing displacement data close to the crack faces (est. 15 %). The approach used to improve results in this study was to replace missing data in the crack region with interpolated data. This improved the overall accuracy (all noise sources considered) to within 5 %.

Experimental validation was performed on PMMA single-edge cracked specimens subjected to mixed loading using a modified Arcan fixture. Decomposed SIFs from specimens loaded in pure mode I, II and III, and mixed loading showed good agreement with mode I fracture toughness values obtained from compact tension tests (ASTM E1820) and mode II and III values extrapolated from literature.

This approach is recommended for the experimentalist who requires simple and efficient calculation of separate mode I-III energy release rates or stress intensity factors, where displacement fields are not limited to those resolved by DIC.

- [1] T. H. Becker, M. Mostafavi, R. B. Tait, and T. J. Marrow, "An approach to calculate the J -integral by digital image correlation displacement field measurement," *Fatigue Fract. Eng. Mater. Struct.*, vol. 35, no. 10, pp. 971–984, Oct. 2012.
- [2] M. Stern, E. B. Becker, and R. S. Dunham, "A contour integral computation of mixed-mode stress intensity factors," *Int. J. Fract.*, vol. 12, no. 3, pp. 359–368, Jun. 1976.
- [3] S. R. McNeill, W. H. Peters, and M. a. Sutton, "Estimation of stress intensity factor by digital image correlation," *Eng. Fract. Mech.*, vol. 28, no. 1, pp. 101–112, Jan. 1987.
- [4] H. Ishikawa, H. Kitagawa, and H. Okamura, "J-Integral of mixed mode crack and its application," in 3rd International Conference on Mechanical Behaviour of Materials, 1980, pp. 447–455.

- chapter 3 -



MS3: Cracking due to coupled processes, including durability mechanics and hydraulic fracture



# Stability of distributed hydraulic fractures against localization: A fundamental problem in fracking of gas shale

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The recent advances in hydraulic fracturing of oil and gas bearing rocks, aka "fracking", have been nothing less than astonishing. These advances have made the US self-sufficient in energy for the first time in a half century. However, while the horizontal drilling and fracking technology has become extremely advanced and sophisticated, the mathematical modeling of interactive fracture mechanics and fluid mechanics aspects of shale fracking is not yet understood. It needs to be, if the current rate gas recovery from the shale strata, which is only about 5% to 15%, is ever to be exceeded. This contribution proposes presents a multi-physics approach to 3D modeling of fluid-driven propagation of a vast network of cracks and open joints in shale, a material of extremely low permeability. The complex nonlinear and anisotropic mechanical behavior of shale is captured by means of a microplane model. Because the crack spacing must be only about 10 cm, according to the current has recovery rate, the fracture of shale analyzed a softening damage in a smeared way by the crack band model in which spurious localization of softening is prevented by a material characteristic length. 3D nonlinear equations governing the Poiseuille flow of compressible cracking fluid through the cracks whose opening and length is controlled by fracture mechanics are formulated. Preliminary numerical simulations of hydraulic fracturing starting from perforations in a horizontal borehole are presented and discussed.

References: Zdenek P. Bazant, Marco Salviato, Viet T. Chau, Hari Visnawathan, and Aleksander Zubelewicz (2014). "Why Fracking Works", ASME J. of Applied Mechanics 81 (Oct.), 101010-1---101010-10.

# Utilization of Kinetic Model on the Expansion of Modified AMBT

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Alkali-silica reaction (ASR) is one of the most recognized deleterious phenomena in concrete and has been a major concern since its discovery in 1940s. The reaction which occurs between reactive silica or silicates present in some aggregates and alkalis of Portland cement produces an alkali-silica gel that expands in the presence of moisture resulting in concrete cracks, spalling and other deterioration mechanisms.

The purpose of this study was to utilize the kinetic model over the mortar expansion data for the 0.5N NaOH to evaluate the alkali-silica reactivity (ASR) of ten aggregate groups. In order to serve the stated purpose, linear expansion of mortar bars made with each aggregate for the 1.0N and 0.5N NaOH solutions was measured, and previously suggested ASR kinetic model, known as Kolmogorov-Avrami-Mehl-Johnson (K-A-M-J) model, was fitted with the expansion data over the test duration of 14 and 28 days. The failure limits for the 0.5N NaOH at 14 and 28 days were developed from the ASR kinetic model to better classify the aggregates into innocuous and reactive. Compared to the previous model, the proposed failure limits showed a better correlation with the findings obtained from the expansion limits in evaluating alkali-silica reactivity of the investigated aggregates.

Keywords: Alkali-silica reactivity, Mortar bar, Expansion, Kinetic model, Test durations.

# **Lattice Modelling of Hydraulic Fracture**

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Characterizing the path of the hydraulic fracture in a heterogeneous medium and the change of the transport properties of this medium is one of the challenges of current research on hydraulic fracturing.

A 2D lattice hydro-mechanical model is used to describe hydraulic fracturing. Coupling is achieved with a staggered scheme that comprises two dual lattices, one for the mechanical problem and another one for fluid flow. In the mechanical problem [1], natural joints are added. They are represented introducing elements with a Mohr-Coulomb behavior. In the fluid problem, Poiseuille flow is considered in each lattice element. A local permeability, which depends on the viscosity of the fluid and on the lattice geometry (opening between two solid parallel planes), is introduced. Macroscopically, it yields a Darcy-type equation in the case of a homogeneous material. The action of fluid pressure on the solid skeleton is represented using Biot's theory. The effect of cracks on fluid flow is captured by increasing the local permeability according to the increase of relative displacements in the dual mechanical lattice element. This increase results from the non-linear response of the skeleton.

First, we present the propagation of fracture restricted to the mechanical case. The influence of a natural joint crossed by the fracture is analyzed with the help of a parametric study. Then, the hydro mechanical coupling is introduced. Results concerning characteristic length and hydraulic fracture behavior are discussed. The size of the area concerned by the change of transport properties, the stimulated reservoir volume, with respect to permeability, is analyzed. This parameter is important in order to evaluate the efficiency of the stimulation by hydraulic fracture in the context of tight hydrocarbon reservoirs.

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# References

 [1] Peter Grassl, David Grégoire, Laura Rojas Solano, and Gilles Pijaudier-Cabot. Meso-scale modelling of the size effect on the fracture process zone of concrete. International Journal of Solids and Structures, 49(13):1818–1827, 2012.

# Numerical modeling of hydraulic fracturing under the influence of natural faults.

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The production rate of oil and gas from unconventional reservoirs, such as shales and tight sandstones, largely depends on the successful creation of hydraulic fractures. The created fracture network increases the conductivity of the reservoir making the production of shale gas economical beneficial. An important aspect of the generation of the fracture network is the interaction between the created hydraulic fractures and natural fractures that are already present in the reservoir [1].

We use our two-dimensional enhanced local pressure model as starting point of this investigation. This model, which can be considered as an extension of the partition of unity based representations, has been developed for hydraulic fracturing simulations [2]. The pressure in the fracture is described by an additional set of degrees of freedom. The pressure gradient due to fluid leakage near the fracture surface is reconstructed analytically, based on Terzaghi's consolidation solution. With this numerical formulation we ensure that all externally applied fluid flow goes exclusively in the fracture. The necessity to use a fine mesh near the fracture to capture the pressure gradient is therefore avoided. Bulk poroelasticity is based on Biot Theory and the fracture process is governed by a cohesive traction separation law. We extend the model to accommodate multiple interacting cracks.

In this contribution we will show hydraulic fracture propagation and nucleation due to fluid injection in a shale rock. By including natural fractures to the initial configuration, we will investigate their influence on the propagation path of a hydraulic fracture.

- [1] D. Taleghani and J.E. Olson. How Natural Fractures Could Affect Hydraulic-Fracture Geometry. *SPE Journal*, 19(1), 161-171. 2014.
- [2] E.W. Remij, J.J.C. Remmers, J.M. Huyghe and D.J.M. Smeulders. The enhanced local pressure model for the accurate analysis of fluid pressure driven fracture in porous materials. *Computer Methods in Applied Mechanics and Engineering*, 286, 293-312, 2014.

# 3D analysis of hydraulic fracture with zerothickness interface elements, comparison with GDK and PKN analytical methods

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The analysis of discontinuities in geological modelling requires specialized numerical techniques. In the present work, zero-thickness interface elements of the Goodman type are used in the context of the FEM. Due to the potentially strong interaction between the solid and fluid phases, a fully coupled (monolithic) hydromechanical formulation [1] is proposed for the analysis of hydraulic fracture.

One of the key points of current investigation is the mechanical constitutive law. In this paper an elastoplastic law which incorporates concepts of fracture mechanics is selected. The history variable controlling the behaviour of interface is formulated in terms of energy dissipated in fracture process, which allows us to introduce the fracture energy spent associated to mode I and mode IIa ( $G_F^I$  and  $G_F^{IIa}$  respectively) as model parameters [2]. Concerning the hydraulic problem, the formulation decomposes the flow in two components along and across the discontinuity, with the corresponding longitudinal and transversal conductivity parameters, which in turn may be affected significantly by the fracture opening (cubic law).

The simulations presented in this paper are an extension of previous work presented by the authors concerning the propagation of a single fracture in 2D [3]. In the current paper, the analysis is focused on the 3D extension of the analysis of a single fracture. The calculations performed include two cases which are compared with previous analytical and numerical solutions [4]. The first example consists of a horizontal layer of 1m of thickness with a line-like distributed flow injection, with the purpose of simulating as close as possible the standard GDK formula [5]. The second case involves a cubic block of 80 m side where three sets of zero-thickness interface are displayed (one over the vertical mid-plane and two sets located at each contact between the horizontal layers) and it is compared with the classical PKN solution [5]. The results obtained in both cases show a good agreement with the formula predictions and with other existing numerical solutions.

- [1] J.M. Segura and I. Carol. "Coupled HM analysis using zero-thickness interface elements with double nodes. Part I: theoretical model, and Part II: verification and application". Int. J. Numer. Analyt. Meth. in Geomechanics, 32(18),2083– 2123, 2008.
- [2] I. Carol, P. Prat and C.M. López. "A normal/shear cracking model. Application to discrete crack analysis". ASCE J. Engrg. Mech., Vol 123(8), 765-773, 1997.
- [3] D. Garolera, I. Aliguer, J.M. Segura, I. Carol, M.R. Lakshmikantha and J. Alvarellos. "Hydro-mechanical coupling in zero-thickness interface elements, formulation and applications in geomechanics". Proceedings of EUROCK 2014, CRC Press, 1379-1384, 2014.
- [4] T.J. Boone and A.R. Ingraffea. "A numerical procedure for simulation of hydraulicdriven fracture propagation in poroelastic media" Int. J. for Numer. Analyt. Meth. in Geomechanics, 14, 27-47, 1990.
- [5] C.H. Yew. Mechanics of hydraulic fracturing. Gulf Publishing Co., Houston, 1997.

# Hydrogen-assisted crack propagation modelling with a cohesive zone model approach

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Since the pioneering work by Barenblatt and Dugdale in the 1960s, cohesive zone models (CZM) have been widely used. Here our aim is to propose a model capable to predict the fatigue crack propagation under high pressure gaseous hydrogen environment. For this purpose, a CZM dedicated to cracking under monotonic as well as cyclic loadings has been implemented in the ABAQUS finite element code. CZM describes the area of the crack tip - called "process zone" - by a traction-separation law (TSL), that is a relationship between the cohesive stress vector (T) and the displacement jump between the lips of the crack ( $\delta$ ). A specific traction-separation law, adapted to describe the gradual degradation of the cohesive stresses under cyclic loading, and sensitive to the presence of dissolved hydrogen at the crack tip is formulated within the framework of Thermodynamics of Irreversible Processes (TIP). The finite element code ABAQUS has no coupled mechanical-diffusion calculation procedure. However, the mass diffusion equation is analogous to the equation of heat. As a consequence the coupling between mechanical behaviour and diffusion of hydrogen can be modelled using a "coupled temperature-displacement" scheme. The bulk material, surrounding the cohesive zone, is assumed to exhibit an elastoplastic behaviour. Finally the hydrogen diffusion in the bulk is accounted for through the equation proposed by Krom et al. [1].

The simulation results are compared with fatigue crack propagation tests performed on a 15-5PH martensitic stainless steel [2]. The effect of hydrogen on the fatigue crack propagation simulations using the proposed CZM model is consistent with experimental crack growth rates at low hydrogen pressure (0.09 *MPa*). The model predicts qualitatively the loss of resistance caused by the presence of hydrogen at the crack tip. However, the drastic loss of resistance to cracking observed at high hydrogen pressure (9 *MPa*) is underestimated.

- [1] A. Krom, R. Koers, A. Bakker, Hydrogen transport near a blunting crack tip, Journal of the Mechanics and Physics of Solids 47 (1999) 971-992.
- [2] C. Moriconi, G. Hénaff, D. Halm, Cohesive zone modelling of fatigue crack propagation assisted by gaseous hydrogen in metals, International Journal of Fatigue 68 (2014) 56-66.

# A microstructural model of porosity based on brittle damage

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Stimulation of oil or gas fields by means of high hydraulic pressure, i. e., fracking, is a common process used to increase and control oil production in a reservoir. Superposed to compressive stress state found in deep veins, the fracking process produces a diffused zone of damaged material, characterized by a higher porosity which facilitates the production, and only under particular conditions it is possible to observe the localization of the damage with the formation of a dominant crack.

Aim of this work is to describe a multi-scale porous brittle damage material model that, in combination with a coupled hydro-mechanics finite element code, can be used to describe the outcomes of fracking processes. The porous material model stems from an existing finite kinematics brittle damage model [1], which describes the distributed damage of brittle materials as a compressive phenomenon that only occurs when sufficient confinement is present. The damage phenomenon is to be intended as opposed to fracture that happens under tensile stress. The brittle damage model is characterized by particular microstructures, consisting of nested families of equi-spaced cohesive faults, bounding otherwise elastic matrix material, referred to as recursive faults, see Fig. 1.

We propose a linearized form of the recursive-faulting model introduced in [1]. We confine our attention to isothermal processes in brittle materials that behave elastically in the absence of damage. We extend this simplified model to include anisotropy of the matrix and, starting from its particular fault microstructure, we regard the damaged material as a porous medium and derive from kinematic considerations its macroscopic permeability [2, 3, 4]. The geomechanical behavior of the medium is then coupled to the flow of a fluid.

We describe the behavior of the proposed material model under loading conditions that characterize typical deep veins of oil and gas field. In particular, we consider an overall compressive anisotropic stress state, and apply a sudden variation of one of the principal stresses which leads to the formation of one or more faults microstructures. Additionally, we consider the same situation with the presence of a natural set of faults, as often observed in oil or gas shale fields, and estimate the formation of faults due to a change of the



Figure 1: (a) Visualization of the microstructures considered in the brittle damage model, where elastic blocks of matrix material are bounded by nested faults. (b) Schematic of the assumed kinematics of deformation, showing the fault orientation N and the displacement jump  $\Delta$ .

stress state. The model predicts the effective or macroscopic behavior of the material from its elastic and fracture properties, and describes the microstructures underlying the microscopic behavior, accounting for fault nucleation and cohesive and frictional behavior of the faults. These features allow for the prediction of damage patterns, the brittle-to-ductile transition resulting under increasing confinement, and the variation of permeability of the porous medium according to the evolution of the faults.

- [1] A. Pandolfi, S. Conti, and M. Ortiz. A recursive-faulting model of distributed damage in confined brittle materials. *Journal of Mechanics and Physics of Solids*, 54:1972–2003, 2006.
- [2] S. Irmay. Flow of liquid through cracked media. *Bulletin of the Research Council of Istrael*, 1(5A):84, 1937.
- [3] D. T. Snow. Anisotropic permeability of fractured media. *Water Resources Research*, 5(6):1263–1279, 1969.
- [4] J. Bear. *Dynamics of fluids in porous media*. American Elsevier Publishing Company Inc, New York, 1972.

# **3D simulation of interface debonding driven by fluid injection**

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Well integrity, a classical topic in the oil and gas industry. To abandon a well, liquid cement column is put into a cased and cemented wellbore and set cement plug. But increasing of reservoir pressure may lead to fracture initiation and propagation in micro annulus at plug-casing interface.

Three dimensional model of fluid driven fracture [1] has been developed and applied to simulate this process and estimate the conditions of fracture initiation and propagation. The main assumptions behind the model are as follows. Cement is considered as linear elastic isotropic homogenous material, steel casing is assumed to be absolutely rigid. Fracture growth is assumed to be quasistatic, and its propagation is described by series of snapshots. Reservoir fluid is supposed to be incompressible homogeneous and Newtonian. No cement erosion, particles transport and fluid filtration through the cement are taken in to account.

The model consists of three sub-models with each one to describe one of the main sub-processes. Deformation of cement plug is described by 3D elastic equilibrium equations that are solved by Boundary Element Method. Pre stress of cement is supposed to be governed by hydrostatic law. Fluid flow inside the fracture is governed by 2D lubrication equations that are solved using the Finite Element Method. Fracture growth condition for brittle materials based on linear elastic fracture mechanics assumptions is used to describe failure of the cement at its boundary with casing. . Interpolation formula is applied to calculate Stress Intensity Factors that are necessary to obtain speed of fracture propagation.

Validation of the model against the laboratory experiment with aluminium tube and poly(methyl methacrylate) block [2] has been performed. Sensitivity analysis has been carried out to show the influence of pre stress of cement plug, difference of reservoir fluid and cement densities (buoyancy [2]), quality and strength of cement-casing adhesion.

It has been observed that shape of the propagated fracture is stable and does not depend on reservoir pressure, pre stress of cement and initial defect (initial fracture) form. Only fracture propagation speed does. Numerical computations show that complete circumferential debonding does not appear except the case of annular initial defect. Fracture circumferential size reaches the value about 2/3 of the wellbore circuit and further the fracture propagates in longitudinal direction only.

- [1] S.G. Cherny, V.N. Lapin, D.V. Esipov, D.S. Kuranakov, G.R. Aidagulov, O.P. Alekseenko 3D model of early stage of hydraulic fracture evolution. Abstracts of Int. conf. "Advanced Mathematics, Computations and Applications", June 2014, Novosibirsk, Russia
- [2] Lecampion B., Bunger A., Kear J., Quesada D. Interface debonding driven by fluid injection in a cased and cemented wellbore: Modeling and experiments. Int. J. of Greenhouse Gas Control, 18, pp. 208–223 (2013)

# Using the coupled criterion to predict surface cracking induced by oxidation in polymers

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Under the action of air at elevated temperature, an oxide layer forms on the surface of thermoset polymers. Due to oxidation and changes in the macromolecular structure (mainly chain scissions), this layer shrinks and its properties (Young's modulus, tensile strength and toughness) are changing. The shrinkage generates residual stresses setting the outer layers of an oxidised specimen in tension and this mechanism is generally enhanced by an increase of the Young modulus. Depending on several parameters, the tension can be or not sufficient to cause multiple cracking on the surface. Anyway, the measurement of the bending strength is modified either by the surface cracks or at least simply by the residual stress [1].

The well-known Griffith law cannot be used to predict any crack nucleation, it only leads to decide whether a *pre-existing* crack can grow or not. Instead, the coupled criterion [2] can, it allows predicting the nucleation of a new crack in a sound structure made of a brittle material. In the general case the onset of such a crack occurs abruptly, the crack jumps from 0 to a given length. Two conditions must be fulfilled: (i) the tensile stress must exceed the tensile strength all along the expected crack path, (ii) the incremental energy release rate must exceed the mode I toughness of the material.

We propose to analyze surface cracking induced by oxidation using the coupled criterion and to compare the results to experiments carried out on specimens made of PR520 epoxy resin [3]. Once it is determined if the oxidation is enough to trigger surface cracking, the model allows predicting the crack depth and spacing [4]. In the other case, i.e. without surface cracking, the failure load in a bending test can be determined, as well as if the crack is arrested or not in the core of the specimen which is under compression.

The originality of this work owes much to the complexity of phenomena where elastic properties and fracture parameters vary with the degree of oxidation which depends on the duration of exposure of the specimens to the air.

- [1] Y. Pannier, C. Guigon, E. Gauthier, M.C. Lafarie-Frenot. Methodology to characterize the influence of thermal ageing on strength properties of epoxy composite matrices. ECCM16 – 16th European Conference on Composite Materials, Seville, Spain, 22-26 June 2014.
- [2] D. Leguillon. Strength or toughness? A criterion for crack onset at a notch. Eur. J. Mech. A/Solids, 21, 61-72, 2002.
- [3] M. Pecora. Etude de l'influence de l'oxydation sur les mécanismes de rupture des résines époxy. Rapport de stage, Institut P', ISAE/ENSMA, Poitiers, France, 2014.
- [4] D. Leguillon, O. Haddad, M. Adamowska, P. da Costa. Crack pattern formation and spalling in functionalized thin films. ECF20 – 20th European Conference on Fracture, Trondheim, Norway, June 30 - July 4, 2014.

# Coupled Hydro-Geomechanical Simulation of Hydraulic Fracture Propagation in Unconventional Reservoirs Using a Mesh Fragmentation Technique

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The interest in studies related to the hydraulic fracturing process has increased over the last decade, mainly due to the exploitation of unconventional reservoirs, which is growing and becoming more important to the current energy demand, with the estimation of the existence of large shale reserves spread over several countries.

Numerical modelling of such processes is a challenging task because of the complexity of the physics involved, and because of the structurally complicated geometry of the reservoir. The interaction between rock's mechanical properties, in-situ stresses, and heterogeneities such as natural fractures and weak bedding planes is determinant of the induced fracture geometry.

To solve this kind of coupled hydro-mechanical problem, the Finite Elements Method is one of the most versatile and widely used. The present study propose a numerical technique called "mesh fragmentation" [1, 2], that uses solid finite elements with high aspect ratio combining with a proper strain softening constitutive model to reproduce the effects of fractures formation process. This methodology, based on the Continuous Strong Discontinuous Approach [3], consists in introducing these high aspect ratio elements between regular elements of a finite element mesh.

Some advantages of this technique are that it can be easily adapted to standard finite elements programs and no tracking algorithms are necessary to follow the evolution of the fracture. The case studies presented in this paper show the ability of the proposed technique to model hydraulic fracturing propagation in unconventional reservoirs.

**Keywords:** Hydraulic Fracturing, Numerical Simulation, Fragmentation Technique, High Aspect Ratio Elements

- M. Sánchez, O. L. Manzoli, L. J. N. Guimarães. Modeling 3-D Desiccation soil crack networks using a fragmentation technique. Computers and Geotechnics, 62, 27-39, 2014.
- [2] O. L. Manzoli, A. L. Gamino, E. A. Rodrigues, G. K. S. Claro. Modeling of interfaces in two-dimensional problems using solid finite elements with high aspect ratio. Computers & Structures, 94-95, 70-82, 2012.
- [3] J. Oliver, D. Linero, A. Huespe, O. L. Manzoli. Two-dimensional modeling of material failure in reinforced concrete by means of a continuum strong discontinuity approach. Computer Methods in Applied Mechanics and Engineering, 197, 332-348, 2008.

# Peridynamic modeling of pitting corrosion damage

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Corrosion damage is an extremely widespread danger to the durability and safety of structures, and there is considerable interest in understanding this phenomenon. Pitting corrosion can lead to accelerated failure of structural components by perforation, or by acting as an initiation site for cracking. The study of pit growth is the main focus in the community of corrosion modeling. These models have successfully predicted the corrosion rate and some are even able to capture the roughening of the corrosion surface. However, since these models consider that the corrosion reaction only affects the evolution of the metal surface, they cannot capture the changes in the mechanical properties in the layer immediately below the solid/liquid interface. These changes, such as embrittlement induced by corrosion or stress-dependence of the diffusion processes in corrosion, are determining factors in explaining how Stress Corrosion Cracking is triggered and how it progresses in time.

In this presentation we introduce a novel peridynamic (PD) model for the evolution of damage from pitting corrosion. We model the anodic reaction in corrosion processes (in which electroplating is negligible) as an effective diffusion process in the electrolyte/solid system combined with a phase change mechanism using peridynamics:

$$\frac{\partial C(x,t)}{\partial t} = \int_{H} k(x,\hat{x}) \frac{C(\hat{x},t) - C(x,t)}{\|\hat{x} - x\|^2} \mathrm{d}V_{\hat{x}} \quad \|\hat{x} - x\| \le \delta \tag{1}$$

where C(x,t) is the metal concentration at a material point x and at time t.  $\delta$  is the horizon size, a nonlocal scale in peridynamic modeling. In our phase-change model, the microdiffusivity  $k(x,\hat{x})$  is depending on the concentration as follows:

$$k(x,\hat{x}) = \begin{cases} k_s & \text{if } C(x) \text{ and } C(\hat{x}) > C_{\text{sat}} \\ k_l & \text{if } C(x) \text{ and } C(\hat{x}) < C_{\text{sat}} \\ \frac{2k_s k_l}{k_l + k_s} & \text{if } C(\hat{x}) \le C_{\text{sat}} \le C(x) \text{ or } C(\hat{x}) \ge C_{\text{sat}} \ge C(x) \end{cases}$$

$$(2)$$

 $k_s$  and  $k_l$  are micro-diffusivities, corresponding to the solid and liquid diffusivities, respectively. The peridyanmic corrosion model coupled with phase-change allows autonomous movement of the interface. To capture subsurface degradation due to corrosion, we introduce a corrosion damage model based on a stochastic relationship that connects the concentration in the metal to the damage of peridynamic mechanical-bonds that are superposed onto diffusion-bonds. We study convergence of this formulation for diffusioncontrolled regime. The model leads to formation of a subsurface damage layer, seen in



Figure 1: (A)Experimental polarization data from (squares) and prediction for the polarization computed with our PD model (triangles). Using just one experimental data-point in the activation-controlled region for calibrating the model, we predict the entire profile of the polarization curve. (B)The damage progression of 2D pitting corrosion in a metal sample with weak passive film on the top. Only the damaged nodes are shown here. The color represents the metal concentration. Blue corresponding to the concentration smaller than 1000 mol/m<sup>3</sup>, while red means the concentration higher than 6000 mol/m<sup>3</sup>. Since the saturated concentration is 5100 mol/m<sup>3</sup>, we can refer the red nodes as the damaged solid nodes. Our model with calibration can quantitatively predict the experimental pit profile (shown in the bottom panel. The top part of the panel is a top view of the sample, while the bottom part is a side view (cross-section) of the pit).

experiments. We validate results against experiments on pit growth rate and polarization data for pitting corrosion (see Figure 1A). We extend the 1D model to the 2D and 3D, and introduce a new coupled model in which in addition to the concentration-dependent damage we allow material phase change based on damage. This coupled model accounts for broken mechanical bonds that enhance corrosion rate. We predict the pit shape and damage profile in materials (Figure 1B) with microstructural heterogeneities, such as defects, interfaces, inclusions, and grain boundaries [1]. Future applications of this model include stress-corrosion cracking, chemo-mechanical degradation in concrete, etc.

# References

[1] Z. G. Chen and F. Bobaru. Peridynamic modeling of pitting corrosion damage. In review.

- chapter 4 -



# MS4: Ductile fracture, modeling of shear bands and necking



# Numerical simulation of biaxial experiments on stress-state-dependent behavior of ductile metals

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The presentation deals with the effect of stress state on inelastic deformation behavior and damage mechanisms of ductile metals. Biaxial experiments are performed and corresponding numerical simulations will reveal stress states and localization of inelastic deformations in critical parts of the tested specimens.

The underlying framework of continuum damage mechanics is based on the kinematic definition of damage tensors [1]. The phenomenological approach takes into account free energy functions defined in fictitious undamaged and damaged configurations, respectively, leading to elastic constitutive laws which are affected by damage. In addition, a hydrostatic-stress-dependent yield condition and a nonassociated flow rule are formulated in the undamaged configurations adequately describing the plastic behavior of ductile metals. Furthermore, a damage criterion and a damage rule formulated with respect to the damaged configurations characterize onset and evolution of damage. They are based on various numerical calculations on the micro-level studying the deformation and failure behavior of void-containing unit cells [2]. Different branches of the damage condition are considered corresponding to different damage and failure mechanisms depending on stress triaxiality and the Lode parameter. To validate the stress-state-dependent functions in the damage condition and the damage rule new experiments with biaxially loaded specimens up to final fracture are performed. Corresponding numerical simulations based on the proposed continuum model show that the tests cover a wide range of stress states and elucidate stress-statedependent damage and fracture mechanisms in critical parts of the specimens. Furthermore, scanning electron microscope images of the fracture surfaces of the specimens allow validation of the damage and fracture mechanisms on the micro-scale predicted by the numerical simulations.

- M. Brünig, O. Chyra, D. Albrecht, L. Driemeier, M. Alves. A ductile damage criterion at various stress triaxialities. International Journal of Plasticity, 24, 1731-1755, 2008.
- [2] M. Brünig, S.Gerke, V. Hagenbrock. Micromechanical studies on the effect of the stress triaxiality and the Lode parameter on ductile damage. International Journal of Plasticity, 50, 49-65, 2013.

# Unraveling the apparent ductility of martensite: a computational micro-scale analysis

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Modern high-strength steels (Dual-Phase, low-alloyed TRIP) are generally multiphase steels, in which the martensitic phase plays a key role. In a multi-phase steel, the generally brittle martensitic phase clearly improves the strength but it may induce a detrimental effect on the ductility. However, recent literature has shown evidence of ductile fracture of martensite. The mechanism behind this ductile fracture is of great interest, since it may help to considerably improve the ductility of high-strength steels in general. The observed ductility may not be an intrinsic property of a martensitic lath, but rather a composite effect at the subgrain scale, where another phase may be retained. For this reason, the term 'apparent ductility' is used, since the small amounts of retained phases are difficult to observe and identify experimentally.

This contribution follows a computational approach to unravel the apparent ductility of martensite. The proper description of the crystallographic characteristics at the microscale and the identification of small amounts of entrapped phases constitutes the basis of the computational model. A martensite grain reveals a well defined internal heterogeneous (crystalline) substructure, i.e. packets, blocks, laths. Thin layers of retained austenite may also be present between the laths, whereby small volume fractions (5%) of interlath retained austenite may already explain the apparent ductility of martensitic subgrains [1]. A conventional crystal plasticity is used to model austenite as FCC and martensitic laths as BCC phases. The entrapped thin FCC phase is shown to act like a greasy plane on which stiff BCC laths can slide. The role of the orientation relationship between FCC and BCC phases is shown to be of fundamental importance for the observed effect. By means of an upscaling technique, the model has been validated with experimental results on a martensitic steel [1]. By accounting for the presence of interlath austenite, the main features of the experimentally observed deformation behaviour (stress-strain curve, slip activity and roughness pattern) are qualitatively well reproduced by the model. It is also shown that, by neglecting the presence of interlath austenite, the observed experimental stress-strain response is not captured. The presence of interlath retained austenite can remarkably enhance the local deformation of martensite.

#### References

[1] F. Maresca, V.G. Kouznetsova, M.G.D. Geers, Subgrain lath martensite mechanics: A numerical-experimental analysis, *Jnl Mech Phys Solids*, 73(15), 69-83, 2014.

# Molecular dynamics investigation of dynamic effects in fracture of ductile materials

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Because many structural materials comprise two or more phases, fracture often involves growth of cracks from one phase to another. Researchers have hypothesized that dynamic effects play an important role in these processes [1]. For instance, carbon steels undergo brittle transgranular fracture when cracks initiated in second-phase carbides extend dynamically into the primary  $\alpha$ -ferrite. Although these cracks would emit dislocations and become blunted at zero velocity, they may instead remain sharp and run brittly if propagating sufficiently fast [2].

To investigate these phenomena, we simulate crack growth in a bi-material strip using molecular dynamics (MD). We employ a novel interatomic potential in which the elastic properties of the material remain fixed, but the intrinsic ductility of the material can be varied systematically. In this way, complications arising from elastic mismatch at the bi-material interface can be avoided. In the MD simulations, cracks initiate within the brittle material, accelerate, and run dynamically toward the interface. We find that materials that are quasi-statically ductile are also dynamically ductile: that is, the running crack emits, blunts, and arrests at the bi-material interface. This finding is shown to be consistent with a continuum-level analysis of dislocation emission from a steady-state, dynamic crack. We conclude by discussing the implications of these findings for fracture of carbon steels and other two-phase materials.

- T. Lin, A. G. Evans, and R. O. Ritchie. Stochastic modeling of the independent roles of particle size and grain size in transgranular cleavage fracture. *Metallurgical Transactions A*, 18(5), 641–651, 1987.
- [2] I.-H. Lin and R. M. Thomson. Dynamic cleavage in ductile materials. *Journal of Materials Research*, 1(01), 73–80, 1986.

# In situ 3D observation of ductile failure mechanisms in cast iron under shear loading

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The ductile failure behavior of nodular cast iron is well understood for loading under intermediate and high stress triaxiality. This process is usually governed by nucleation, growth and coalescence of voids. In nodular cast iron, the graphite particles debond under tension loading from the metallic matrix and thereby relatively large voids develop. However, there is only little knowlegde about the failure mechanisms under shear loading.

Therefore, *in situ* experiments with large flat shear specimens have been performed at beamline ID15a of the European Synchrotron Radiation Facility (ESRF). By using X-ray laminography for 3D observation [1] the nodular cast iron can be studied during the entire deformation process in a region of interest within the specimen. The mechanisms leading to ductile failure under shear loading such as shear band formation, void shape and void volume evolution are investigated in detail with micrometer resolution. Furthermore, the results are compared to predictions of a shear-modified GTN model [2].

- [1] T.F. Morgeneyer, L. Helfen, I. Sinclair, H. Proudhon, F. Xu, T. Baumbach: Ductile crack initiation and propagation assessed via in situ synchrotron radiation-computed laminography. *Scripta Materialia*, 65(11),1010–1013, 2011.
- [2] K. Nahshon, J. Hutchinson. Modification of the Gurson model for shear failure. *European Journal of Mechanics-A/Solids*, 27(1):1–17, 2008.

# A Homogenization model for porous crystals comprising general ellipsoidal voids

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#### ABSTRACT

Voids originate in the manufacturing process have an important effect on the lifetime of materials and play a important role even on metallic alloys. Indeed, as recently indicated by experimental observations (Srivastava et al. [2012]) at high enough temperatures on tensile specimens, the growth of initially present processing induced voids in a nickel based single crystal superalloy as well as in standard polycrystals played a significant role in limiting creep life. The presence of voids in metals is known to be one of the major causes of ductile failure, as addressed in pioneering works by (Mc Clintock [1968], Rice and Tracey [1969], Gurson [1977]). Most of the studies have been carried out in the case of two-phase material systems comprising an isotropic rate independent matrix phase (metal usually described by von Mises yield criterion) and a voided phase (pores of spherical, spheroidal or arbitrary ellipsoidal shapes).

Far fewer results have been obtained for rate-dependent anisotropic matrix systems, generally based on a phenomenological Hill-type matrix. However, there is no damage ductile growth model for the case of porous single crystals. When these material systems are subjected to external loads impurities fail or decohere leading to the creation of pores, which in turn evolve in size, shape and orientation (Srivastava and Needleman [2012]). This complex evolution of microstructure together with the evolution of the rate-dependant matrix anisotropy is critical in the prediction of the eventual fracture of the specimen under monotonic and cyclic loading conditions.

The purpose of the present work is to develop a model to deal with rate-dependent crystalline matrix phase comprising general ellipsoidal voids that could possibly evolve in shape and orientation, using a variational homogenization framework. The prediction capabilities of the model are compared with finite element computations obtained from unitary and multiple void unit-cells.

#### **RESULTS AND DISCUSSION**

The numerical validation of the model has been carried out through FEM, with periodic geometry (see fig 1).

As an important result depicted in fig 2, the present estimate takes into account the anisotropy of the single crystal and anisotropy of the microstructure, while exhibiting very complex coupling both.



Figure 1: Contour of the total slip for a FCC single crystal with a "multipore" geometry of 40 pores (porosity f = 5%), and several creep exponent (a) n = 1 (b) n = 5 (c) n = 10.



Figure 2: Gauge surface for a porous FCC crystal with elliptical voids. (a) Comparison between the model (MVAR) and the FE results in the  $\overline{\Sigma}_m - \overline{\Sigma}_{eq}$  plane, for a porosity f = 1% and a range of creep exponent n = (1, 2, 5, 10). (b) In the deviatoric plane  $\overline{\Sigma}_{12} - (\overline{\Sigma}_{11} - \overline{\Sigma}_{22})/2$  for a creep exponent n = 10, voids of aspect ratio w = 0.2, orientation  $\psi = 67.5^{\circ}$ , a porosity f = 5%, at different level of pressure.

- A.L. Gurson. Continuum theory of ductile rupture by void nucleation and growth. J. Eng. Mater. Tech., 99:2–15, 1977.
- F. Mc Clintock. A criterion for ductile fracture by the growth of holes. J. App. Mech., 35: 363–371, 1968.
- J.R. Rice and D.M. Tracey. On the ductile enlargement of voids in triaxial stress fields. J. Mech. Phys. Solids, 17:201–217, 1969.
- A. Srivastava and A. Needleman. Porosity evolution in a creeping single crystal. *Model. Simul. Mater. Sci. Eng.*, 20, 2012.
- A. Srivastava, S. Gopagoni, A. Needleman, V. Seetharaman, A. Staroselsky, and R. Banerjee. Effect of specimen thickness on the creep response of a ni-based single-crystal superalloy. *Acta Materialia*, 60:5697–5711, 2012.

# Void growth and coalescence in an irradiated single crystal

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Voids can form in austenitic stainless steels, when the steels are submitted to particle irradiation. In virgin austenitic stainless steels, ductile fracture is known to take place as a result of micrometric void growth and coalescence. In irradiated stainless steels, irradiation-induced voids are small, compared to the grain size. Thereby, one can consider the voids as embedded in a large single crystal matrix. In order to model the damage caused by void growth and coalescence in the irradiated steels, it is necessary to develop a set of constitutive equations at the grain scale, i.e. the scale of single crystal.

In our study, void growth and coalescence in FCC single crystal have been investigated using 3D unit cell finite element simulations. For the simulations, the crystal plasticity based constitutive model, proposed by Tanguy *et al* [1] and Han [2] for austenitic stainless steels neutron irradiated in a mixed spectrum at about 300°C, is used. The material parameters of this model have been identified by the authors for different levels of irradiation damage. A unit cell containing a spherical void is deformed under constant macroscopic stress triaxiality with periodic boundary conditions. The simulations are performed for different crystal orientations and stress triaxialities. The effect of irradiation over void growth and coalescence is studied.



Fig 1: total cumulative slip of a unirradiated single crystal with orientation [100]-[010]-[001] and stress triaxiality T=2.0


Fig 1: total cumulative slip of a single crystal irradiated to 13 dpa with orientation [100]-[010]-[001] and stress triaxiality T=2.0

On the other hand, Han *et al.* [3] have developed a yield potential function, which combines resolved shear stress, von Mises equivalent stress and hydrostatic stress, using a variational homogenization approach for describing the macroscopic yield surface of FCC single crystal containing voids:

$$\Phi_{s} = \left(\frac{\tau_{s}^{2}}{\tau_{s}^{*2}} + \alpha \frac{2f}{45} \frac{\sigma_{eq}^{2}}{\tau_{s}^{*2}}\right) + 2q_{1}f \cosh\left\{q_{2}\sqrt{\frac{3}{20}} \frac{\sigma_{m}}{\tau_{s}^{*}}\right\} - 1 - (q_{1}f)^{2} \stackrel{\text{def.}}{=} 0$$

Combining this yield potential function [3] and crystal plasticity based model of Tanguy *et al.* [1] and Han [2], we derive the constitutive equations by thermodynamic approach for porous single crystal in the finite strain framework. Finite element simulations have been carried out to predict the void growth of porous single crystal under triaxial loading for different lattice orientations. The simulations are then assessed by comparing the results with the 3D unit cell finite element simulations.

Based on the unit cell FE simulations, an effective porosity  $f_*$ , which is a function of actual porosity f, is introduced in our model to describe coalescence as an accelerated process of void growth. It is assumed that when a critical porosity is reached, coalescence process begins by faster void growth.

- B. Tanguy, X. Han, J. Besson, S. Forest, C. Robertson and N. Rupin, International Symposium on Plasticity 2013 and its Current Applications, Nassau, Bahamas, 3-8 January 2013
- X. Han, Modeling of cavity swelling-induced embrittlement in irradiated austenitic stainless steels, Ph.D Thesis, Mines ParisTech, 2012
- X. Han, J. Besson, S. Forest, B. Tanguy and S. Bugat, Internatinal Journal of Solids and Structures, 50 (2013) 2115-2131.

# Micromechanical modeling of ductile fracture mechanisms using a new body-fitted immersed volume method

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At the microscale, metallic materials ductile fracture process is based on nucleation (particles fracture or interface debonding), void growth and coalescence mechanisms. A new finite element (FE) approach based on level set functions and anisotropic mesh adaptation was developed to model such mechanisms for large plastic strain [1, 2]. In order to improve volume conservation and interfaces description a body-fitted immersed method was developed and is detailed in [3, 4] with applications to coalescence.

This new FE approach is used to study the influence of multiaxial and nonmonotonic loading on ductile failure mechanisms. It is shown that different loading paths leading to the same final macroscopic strain activate different nucleation mechanisms and consequently different final void volume fraction. Additionally, even at equal void volume fraction, void morphology is proven to be of important influence on ductility. Special attention is paid to the influence of void arrangements (periodic or random) on void growth and coalescence [3].

In order to calibrate nucleation and coalescence criteria, it is important to compare numerical simulations with experimental results. Synchrotron Radiation Computed Laminography (SRCL) experiments and observations were carried out on nodular cast iron for different loading conditions [5]. A representative volume element (RVE) representing the exact microstructure is meshed using the body-fitted immersed method presented in [3]. Based on SRCL images, Digital Volume Correlation (DVC) [6] can be used to extract the exact boundary conditions that need to be applied on the RVE. The modelling of nodular cast iron failure mechanisms is then carried out at the microscale using these exact boundary conditions and comparison with experimental observations is presented and discussed.

## References

[1] E. Roux, M. Bernacki and P.-O. Bouchard, A level-set and anisotropic adaptive remeshing strategy for the modeling of void growth under large plastic strain, *Computational Materials Science*, 68, 32-46, 2013.

[2] E. Roux, M. Shakoor, M. Bernacki and P.-O. Bouchard, A new finite element approach for modelling ductile damage void nucleation and growth – analysis of loading path effect on damage mechanisms, *Modelling and Simulation in Materials Science and Engineering*, 22:1-23, 2014.

[3] M. Shakoor, M. Bernacki and P.-O. Bouchard, A new body-fitted immersed volume method for the modeling of ductile fracture: analysis of void clusters and stress state effects on coalescence, submitted for publication in *Engineering Fracture Mechanics*.

[4] M. Shakoor, M. Bernacki and P.-O. Bouchard, An adaptive body-fitted monolithic method for modeling the fracture of heterogeneous microstructures, International Conference on Computational Modeling of Fracture and Failure of Materials and Structures (CFRAC2015), 3-5 June 2015, Paris, France.

[5] L. Zybell, M. Horn, M. Kuna, L. Helfen and T.F. Morgeneyer, In situ 3D observation of ductile failure mechanisms in cast iron under shear loading, International Conference on Computational Modeling of Fracture and Failure of Materials and Structures (CFRAC2015), 3-5 June 2015, Paris, France.

[6] T.F. Morgeneyer, T. Taillandier-Thomas, L. Helfen, T. Baumbach, I. Sinclair, S. Roux, F. Hild, In situ 3-D observation of early strain localization during failure of thin Al alloy (2198) sheet, *Acta Materialia*, 69 : 78-91, 2014

## Mesh objective models for ductile fracture based on a damage phase field concept

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The Johnson-Cook (JC) model for ductile failure is simple and phenomenological. Being derived for ductile fracture in metals with the involvement of only few parameters, the model has been shown to work well in many other applications. In spite of its simplicity, it catches the material behavior for large deformation/high speed/temperature applications at a reasonable cost. In addition to its widespread use in commercial software, low cost is an obvious advantage, which makes the model often used in machining simulations of metal cutting, cf. [1]. A major drawback is that the JC–material model exhibits mesh size dependence which is known from orthogonal machining simulations. In this context many researchers have proposed remedies of various type, we mention: [2], [3] using the concepts of a damage phase field, [4] using special element enhancement of the FE–kinematics.

In this context, combined with concepts of a damage phase field (without gradient enhancement), we consider the mesh objective element removal and progressive damage models. Relating to the ideas of ref. [3], a central point in the modeling concerns the handling of the maximum energy dissipation rate principle for scalar damage evolution involving the total dissipation. To control evolution of the damage, both inelastic continuous deformation and localized deformation due to damage evolution are thus considered to define a total damage driving energy  $A_T$  in the damage criterion. In order to link the continuum damage to the fracture modeling some concept of the phase field formulation of e.g. [3] are exploited. Based on the assumption of a localized damage field, a mesh objective formulation is obtained in terms of scaling factor from the element diameters of the reference FE–element mesh. In turn, the JC–model parameters are considered calibrated with respect to the reference mesh.

We propose in this paper two FE–mesh objective technologies, enhancing the JC– model for ductile fracture using the concept of scalar damage. Regardless of the damage model used, a fracture state is achieved at the Gauss point level when the accumulated effective plastic strain approaches the fracture strain of the JC–failure model. For the element removal model an instantaneous damage evolution is achieved when the total damage driven energy,  $A_T$ , equals the scaled release energy at the Gauss point level. When this is obtained the damage criterion is then met and the element is removed, corresponding to full stress relaxation. For the progressive damage model a similar evaluation of the damage is conducted, although the damage evolution is progressive. From the phase field concept, in this case the mesh objective scaling depends on the parameters of the progressive damage evolution.

The proposed models have been implemented in a large deformation setting in Matlab. To illustrate the advantages using the mesh objective enhanced models a shear test of a plate is investigated at plane strain conditions. A comparison is made between mesh objective enhanced damage models and damage models without any objective enhancement. Please note the convergence of the mesh dependence for the models in Figure 1



Figure 1: Reaction force vs displacement response using the mesh objective models of progressive damage and element removal

- G. Ljustina, R. Larsson and M. Fagerstrom. Microstructure level Machining Simulation of Compacted Graphite Iron. *Finite elements in analysis and design*, 80:1–10 (2014).
- [2] GA. Francfort, JJ. Marigo, Revisiting brittle fracture as an energy minimization problem. *Journal of the Mechanics and Physics of Solids*, 46:1319–1342 (1998).
- [3] C. Miehe, M. Hofacker, L. Schanzel, F. Aldakheel, Phase field modeling of fracture in multi-physics problems. Part II. Coupled brittle-to-ductile failure criteria and crack propagation in thermo-elastic-plastic solids, Comput. Methods Appl. Mech. Engrg. (2014).
- [4] A. Huespe, A. Needleman, J. Oliver, Sanchez, A finite strain, finite band method for modeling ductile fracture, International Journal of Plasticity 28:53–69 (2012).

# Localization and damage interactions in Al-alloy sheets: *in situ* 3D measurements

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Up to now, the interactions between plasticity and damage remain obscure when the fracture of ductile sheet materials is studied (e.g., flat to slant transition). The fundamental question becomes: Which mechanism (i.e., damage or plasticity) is responsible for localized phenomena leading to the final failure?

In this work, damage and strain interactions have been assessed *in situ* and in the material bulk at the micrometer scale during ductile tearing. The region of interest was located at about 1 mm ahead of notched 1 mm thick plates made of aluminum alloys. This assessment has been achieved by combining *in situ* synchrotron laminography [1] and digital volume correlation using the material microstructure contrast as markers [2]. Synchrotron laminography is a technique specifically developed for three dimensional imaging of volumes of interest in laterally extended sheet specimens. The *in situ* data allowed the evolution of individual voids to be assessed at different positions on the slant fracture surface with different associated stress and strain histories [3]. A link between the measured strain fields and void evolution is made. Slanted strain localization bands are found to precede the onset of damage for the investigated materials [4].

- [1] Morgeneyer T.F., Helfen L., Sinclair I., Proudhon H., Xu F., Baumbach T., Ductile crack initiation and propagation assessed via in situ synchrotron radiation computed laminography, Scripta Materialia, 65, 1010-1013, 2011
- [2] Morgeneyer, T. F., Helfen, L., Mubarak, H. and Hild, F. 3D Digital Volume Correlation of Synchrotron Radiation Laminography Images of Ductile Crack Initiation: An Initial Feasibility Study. Experimental Mechanics 53(4), 543-556, 2013
- [3] Ueda T., Helfen L., Morgeneyer T.F., In-situ laminography study of three-dimensional individual void shape evolution at crack initiation and comparison with GTN-type simulations. Acta Materialia, 78, 254–270, 2014

[4] Morgeneyer, T.F., Taillandier-Thomas, T., Helfen, L., Baumbach, T., Sinclair, I., Roux, S. and Hild, F. In situ 3-D observation of early strain localization during failure of thin Al alloy (2198) sheet, Acta Materialia 69, 78–91, 2014.

# Porous materials with Mohr-Coulomb matrix: theoretical investigation and numerical validation

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The aim of this study is to investigate the specific effects of Mohr-Coulomb matrix on the strength of ductile porous materials by developing a kinematics limit analysis approach for the theoretical part and both static and kinematic numerical bounds. We first focus on the theoretical formulation of a macroscopic strength criterion for porous Mohr-Coulomb materials. To this end, following Gurson (1977) approach, we consider a hollow sphere model, but here with a rigid perfectly plastic Mohr-Coulomb matrix, subjected to axisymmetric uniform strain rate boundary conditions. Taking advantage of an appropriate family of three-parameter trial velocity fields accounting for the specific plastic deformation mechanisms, we then provide a solution of the constrained minimization problem required for the determination of the macroscopic dissipation function. The macroscopic strength criterion is then obtained by means of a Lagrangian method combined with Karush-Kuhn-Tucker conditions. Owing to the specificities of the Coulombmatrix (dependence on all the stress invariants), a careful analysis and discussion of the plastic admissibility condition (which involves the knowledge of the sign of the principal strain rates) is required. The proposed procedure leads to a parametric closed-form expression of the macroscopic strength criterion. The later explicitly shows a dependence on the three stress invariants. In the special case of a friction angle equal to zero, the established criterion reduced to recently available results for porous Tresca materials. For completeness, the macroscopic plastic flow rule and the voids evolution law are fully detailed.

For validation purpose, a complete numerical Limit Analysis of a hollow sphere model having a Coulomb solid matrix is proposed. Brief backgrounds and fundamental of the static and kinematic approaches in the context of numerical limit analysis are first recalled. We then present the hollow sphere model, together with its axisymmetric FEM discretization. After an adaptation of a previous static code, an original mixed (but fully kinematic) approach dedicated to the axisymmetric problem was elaborated with a specific quadratic velocity field associated to the triangular finite element. Despite the less good conditioning inherent to the axisymmetric modelization, the final conic mixed code

appears very efficient, allowing to take into account numerical meshes highly refined. After a first validation in the case of spherical cavities and isotropic loadings, for which the exact solution is known, numerical lower and upper bounds bounds of the macroscopic strength are provided. These are used to assess and fully validate the results provided by the theoretical analysis. Effects of the the friction angle as well as that of the porosity are illustrated.

Considering axisymetric loadings ( $\Sigma_x = \Sigma_y$ ), the generalized loading vector is composed, in a Cartesian coordinate system (with orthonormal basis ( $\underline{e}_x, \underline{e}_y, \underline{e}_z$ )), by:

$$\Sigma_m = \frac{1}{3} \left( 2\Sigma_x + \Sigma_z \right), \qquad \Sigma_{gps} = \Sigma_x - \Sigma_z \tag{1}$$

On figure 1 are displayed the effects of material friction angle. The results are normalized by the matrix cohesion c. In general, a very remarkable agreement is obtained between theory and the numerical bounds. Significant asymmetries of the obtained surfaces are also observed, more particularly when the friction angle increases and the porosity is fixed: in particular, the tension and the compression hydrostatic points are reduced and increased respectively with an increase of  $\phi$ ). Note that the asymmetry of the obtained strength surfaces is a signature of the dependence of the macroscopic criterion on all three stress invariants (and notably the third stress invariant).



Figure 1: Porous Mohr-Coulomb materials: comparison of the predictions by the proposed strength criterion with the numerical bounds for a porosity: f = 0.01 and different values of friction angle  $\phi = 5$  degrees,  $\phi = 10$  degrees and  $\phi = 25$  degrees.

# A bipotential-based approach of ductile porous materials having a non-associated matrix: theory and numerical assessment.

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In the framework of kinematics limit analysis theory, as implemented by Gurson (1977), Guo and al. (2008) have proposed a macroscopic plastic model for ductile porous materials with pressure-sensitive dilatant matrix obeying to the normality law (associated material). The aim of the present study is to extend this class of models to geomaterials for which the solid matrix obeys to a non-associated plastic flow rule (dilatancy angle is different from the friction one). In this case, the normality property fails and classical limit analysis cannot be applied. The usual presentation of non associated plasticity is based on a yield function and a plastic potential (to represent the flow rule). Although it is intensively used in the literature, this approach is in fact not very relevant in formulating variational methods required by limit analysis procedures. For this reason, the second last author had proposed (see De Saxce et al.,1991; 1992) a suitable modeling framework based on the bipotential, a function of both dual variables, the plastic strain rate and stress tensors.

On this ground, we deduce in the present study a macroscopic model for porous materials with a nonassociated matrix by using a bipotential-based variational approach. The proposed procedure is implemented for the hollow sphere model by adopting simple but suitable trial velocity field and trial stress field. The macroscopic yield function, obtained in a parametric form, is porosity f and dilatancy angle dependent. As in Maghous et al. (2009), the non-associated character of the macroscopic flow rule is shown.

We aim now at illustrating the macroscopic criterion both in associated and non-associated cases. The matrix pressure sensitivity is characterized by the friction angle  $\phi$  and the dilatancy one  $\psi$  for the Drucker-Prager model. We provide here on figure 1 illustration of the established criterion for a porosity f = 0.2 and friction angle  $\phi = 30^{\circ}$ . The corresponding associated case  $\psi = \phi = 30^{\circ}$  is denoted AC, while two non-associated cases are considered; they are respectively defined by dilatancy angles  $\psi = 15^{\circ}$  (NAC1) and  $\psi = 5^{\circ}$  (NAC2).

For hydrostatic loadings (traction and compression), the non-associated cases provide the same predictions as that of the associated one (see Guo et al., 2008). This observation is in full agreement with available theoretical and numerical results (see Cheng et al. 2015). Furthermore, the non-associated cases show in general different yield loci with respect to

the associated one: as expected, the yield surface for a non associated case is lower than the associated one. Note that a decrease of the dilatancy angle leads to a weaker strength, the difference between the cases  $\psi = 15^{\circ}$  and  $\psi = 5^{\circ}$  being however slight. These results has been assessed by means of numerical FE results carried out in the context of non associate plasticity (see Cheng et al. 2015 for more details). They are supported by the numerical data.

#### References



Figure 1: Comparison of yield surfaces between the associated case (denoted AC) with dilatancy angle  $\psi = 30^{\circ}$  and two non-associated cases (denoted NAC1 and NAC2) with  $\psi = 15^{\circ}$  and 5°, respectively. Porosity: f = 0.2; friction angle  $\phi = 30^{\circ}$ .

L. Cheng, Yun Jia, A. Oueslati, G. de Saxce, D. Kondo. Bipotential-based limit analysis and homogenization of non-associated porous plastic materials. *Journal of the Mechanics and Physics of Solids*, 77, 1-25, 2015.

G. de Saxce, Z.Q. Feng. New inequality and functional for contact friction: The implicit standard material approach. *Mechanics of Structures and Machines*, 19, 301325, 1991.

G. de Saxce. Une generalisation de l'inegalite de Fenchel et ses applications aux lois constitutives. *C.R. Acad. Sci.*, Paris, Ser. II, 314, 125-129, 1992.

T.F. Guo, J. Faleskog, C.F. Shih. Continuum modeling of a porous solid with pressuresensitive dilatant matrix. *Journal of the Mechanics and Physics of Solids*, 56, 2188-2212, 2008.

A.L. Gurson. Continuum theory of ductile rupture by void nucleation and growth- Part I: yield criteria and flow rules for porous ductile media. *ASME J. Engng. Mat. Technol.*, 99, 2-15, 1977.

S. Maghous, L. Dormieux, J.F. Barthelemy. Micromechanical approach to the strength properties of frictional geomaterials. *European Journal of Mechanics A/Solids*, 28, 179-188, 2009.

## New three-dimensional strain-rate potentials for isotropic porous metals: role of the plastic flow of the matrix

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At present, modeling of the plastic response of porous solids is done using stressbased plastic potentials (see [1]-[2]). To gain understanding of the combined effects of all invariants for general three-dimensional loadings, a strain-rate based approach appears more appropriate. In this paper, for the first time strain rate-based potentials for porous solids with Tresca and von Mises matrices are obtained. The dilatational response is investigated for general 3-D conditions for both compressive and tensile states using rigorous upscaling methods. It is demonstrated that the presence of voids induces dependence on all invariants, the noteworthy result being the key role played by the plastic flow of the matrix on the dilatational response (see Fig.1). If the matrix obeys the von Mises criterion, the shape of the cross-sections of the porous solid with the octahedral plane deviates slightly from a circle, and changes very little as the absolute value of the mean strain rate increases. However, if the matrix behavior is described by Tresca's criterion, the shape of the cross-sections evolves from a regular hexagon to a smooth triangle with rounded corners. Furthermore, it is revealed that the couplings between invariants are very specific and depend strongly on the particularities of the plastic flow of the matrix (see also [3]).

- [1] O. Cazacu, B. Revil-Baudard, R.A. Lebensohn, M. Garajeu, On the combined effect of pressure and third invariant on yielding of porous solids with von Mises matrix, *Appl Mech-T Asme*, Vol. **80**, 064501
- [2] O. Cazacu, B. Revil-Baudard, N. Chandola, D. Kondo, New analytical criterion for porous solids with Tresca matrix under axisymmetric loadings, *Int J Solids Struct*, Vol 51, pp 861-874, 2014.
- [3] B. Revil-Baudard and O. Cazacu, Role of the plastic flow of the matrix on yielding and void evolution of porous solids: comparison between the theoretical response of porous solids with Tresca and von Mises matrices, *Mech Res Commun*, Vol **56**, pp 69-75, 2014.
- [4] B. Revil-Baudard and O. Cazacu, New strain-rate plastic potentials for porous metals, *Int. J. Plasticity*, 60, 2014, 101-117.

## **Plasticity-damage couplings in Titanium alloys**

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Low symmetry metallic materials (e.g. titanium, etc.) display deformation and failure properties that are quite different from that of typical materials with cubic crystalline structure (aluminum, steels, etc). Rolled or extruded products exhibit a strong anisotropy and very pronounced difference in yielding and work-hardening evolution between tension and compression loadings. In this paper, microstructure sensitive models for description of the plastic deformation of Ti are presented. These models are based on a detailed multi-scale characterization conducted in order to identify the physical mechanisms at the single crystal level and their effects on the macroscopic response. The effects of texture evolution are explicitly modeled. It is demonstrated that damage accumulation depends strongly on the specificities of the plastic deformation of the matrix, namely its tension-compression asymmetry. Moreover, very recent results concerning plasticity-damage couplings in Ti materials are presented.

- [1] Cazacu, Plunkett and Barlat. 2006. Orthotropic yield criterion for hexagonal closed packed materials. Int. J. Plasticity, volume 22: 1171-1194.
- [2] Stewart and Cazacu. 2009. Plastic potentials for porous aggregates with the matrix exhibiting tension-compression asymmetry. J. Mech. Phys. Solids., volume 57: 325-341.
- [3] Stewart and Cazacu. 2011. Analytical plastic potential for an anisotropic material containing spherical voids and exhibiting tension-compression asymmetry. Int. J. Solids Struct., volume 48: 357-373.
- [4] Revil-Baudard and Cazacu. 2013. On the effect of the matrix tension-compression asymmetry on damage evolution in porous plastic solids, European Journal of Mechanics - A/Solids, volume 37: 35-44.

# Damage evolution law based on the micromechanical of defects and coupled in an unconventional yield surface

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The ductile fracture prediction is a challenger studied by researchers around the word and an important subject of interesting for industrial sectors, such as automotive, aeronautic and offshore exploration. However, despite years of studies, many questions remain open, for example, how to describe correctly the stress and strain fields under wide range of stress triaxiality or in presence of multiaxial and non-proportional loading histories. Furthermore, others parameters of studies are in evidence, such as the determination of the level of displacement at fracture as well as the potential site to fracture initiation. In this sense, the introduction of important effects both in the damage evolution law and/or in the definition of the yield surface have been able to promote improvements in the constitutive formulations. The third invariant and stress triaxiality effects, dependence of multiple calibration points and multiscale damage approaches are important effects responsible for those improvements. In this contribution a damage evolution law based on the micromechanical of defects is coupled in an advanced yield surface both of them dependents of the third invariant and multiple calibration points. The new constitutive formulation is implemented in an academic finite element development and its robustness is performance by different specimens under wide range of loading histories. The accuracy of the new proposition is evaluated by the numerical prediction of the displacement at fracture, prediction of the fracture onset and the determination of the stress and strain fields under complex load.

- Gao, X., Zhang, T., Zhou, J., Graham, S.M., Hayden, M., Roe, C., 2011. On stressstate dependent plasticity modeling: significance of the hydrostatic stress, the third invariant of stress deviator and the non-associated flow rule. International Journal of Plasticity 27 (2), 217–231;
- [2] Mamiya, E. N., Malcher, L., Reis, F. J. P., Andrade, F. X. C., Cavalheiro, J. V. S., 2014. Elasto-plastic model based on the third invariant of the deviatoric stress tensor: Monotonic and cyclic loading application. 11th. World Congress on Computational Mechanics (WCCM XI);

[3] Malcher, L.; Andrade Pires, F.M.; César de Sá, J.M.A., (2013). An Extended GTN Model for Ductile Fracture under High and Low Stress Triaxiality. International Journal of Plasticity, v. 54, p. 193-228.

# Use of a continuum damage model with the improved Lemaitre's damage evolution law to estimate fatigue life

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The present contribution proposes the use of an improved continuum damage model in order to estimate low cycle fatigue life under both proportional and non-proportional loading conditions. This approach is based on the modified Lemaitre's damage evolution law proposed by Malcher & Mamiya [1], regarding the denominator of damage function dependent on both stress triaxiality and normalized third invariant. In order to estimate fatigue life, the non-linear kinematic hardening law proposed by Armstrong & Frederick [2] has been assumed as well as the crack closure effect [3]. An implicit numerical integration scheme is suggested and implemented in an academic finite element framework. Numerical results are compared with experimental data available in literature and parameters as fatigue life, evolution of damage variable and stress/strain fields are widely analysed.

- [1] Malcher, L., Mamiya, E.N. (2014). An Improved Damage Evolution Law Based on Continuum Damage Mechanics and its Dependence on Both Stress Triaxiality and the Third Invariant. International Journal of Plasticity, v. 56, p. 232-261, 2014.
- [2] Armstrong, P. J., Frederick, C. O. (1966). A mathematical representation of the multiaxial Bauschinger effect. Report RD/B/N731, CEGB, Central Electricity Generating Board, Berkeley, UK.
- [3] Desmorat, R., Kane, A., Seyedi, M., Sermage, J.P. (2007). Two scale damage model and related numerical issues for thermo-mechanical high cycle fatigue.

# High and low stress triaxiality GTN-based ductile failure model

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Many of the available coupled damage models predict correctly fracture location for certain stress states, namely for moderately and high triaxiality states, but very often fail to give the appropriate answers to low triaxiality stress states, particularly when shear effects dominate. Furthermore, their parameter identification is strongly influenced by the calibration point.

This is the case of one of the most used models, the GTN (Gurson–Tvergaard– Needleman) model. Because it is based on Gurson theory that assumes that the spherical void growth is the predominant mechanism of ductile failure, it is adequate for moderate and high levels of stress triaxialities where this mechanism dominates. However when shear is predominant the deformation failure localization is mainly due to void rotation and distortion and the model does not perform well.

Therefore an enhancement on the GTN model is implemented to provide a better prediction at low level of stress triaxiality states. Two independent damage parameters are considered as internal damage variables: one similar to the Gurson's original model and associated with the spherical void growth and a second one associated with void rotation and distortion. The first one is a function of the stress triaxiality. The second one depends on the equivalent plastic strain, Lode angle and stress triaxiality. It is independent of the volume void fraction and a different nucleation mechanism is assumed. The two damage parameters are included in the yield function so that they affect separately the hydrostatic stress and the deviatoric stress effects.

The parameters associated with the two failure mechanisms are calibrated with the results obtained with two different material specimens at high and low stress triaxiality conditions. Numerical solutions are compared with experimental results for different deformation paths under mixed loading conditions.

## References

 L. Malcher, F.M. Andrade Pires, J.M.A. Cesar de Sa. An extended GTN model for ductile fracture under high and low stress triaxiality. International Journal of Plasticity 54 (2014) 193–228.

# Ductile damage analysis by particles tracking in large tomographic dataset

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Ductile fracture has been studied extensively over the last five decades. It is composed of three steps: nucleation of cavities, growth of these nucleated cavities and coalescence leading to the final fracture. These damages in ductile materials can nowadays be imaged in 3D using different methods at different scales. The most commonly used in the field of ductile damage is probably the X-Ray Computed Tomography (XRCT) because of its multimaterial, multiscale and non destructive character. It can be coupled with in situ loading cells to follow in 3D the evolution of the microstructure under different loading-paths. One can then focus on imaging and understanding each of the three steps of ductile damage as for example in the case of tension tests on aluminum alloys or TA6V samples.

To go deeper in the analysis on this large amount of data, a specific algorithm has been implemented and further developed to track and follow individually each cavity [1] (few thousands), allowing more accurate determinations of the nucleation/growthrate/coalescence as function of the macroscopic strain applied, the void environment... and showing a large deviation from the standard laws.

This presentation will illustrate this new analysis technique and how we have used it in recent studies in order to understand the damage process of heterogeneous materials. Very high resolution and high acquisition rates, two new tendancies in this field, will also be illustrated.

## References

 L. Lecarme, E. Maire, K.C. A. Kumar, C. De Vleeschouwer, L. Jacques, A. Simar and T. Pardoen. Heterogenous void growth revealed by in situ 3-D X-ray microtomography using automatic cavity tracking. *Acta Materialia*, 63, 130–139, 2014. - chapter 5 -



# MS5: Dynamic fracture, fragmentation and impact



# Dynamic fracture of thin shells: an X-FEM approach

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This paper is devoted to the presentation of an X-FEM element for thin shell dynamic crack propagation simulation. The element is based on the  $Q4\gamma 24$  4 nodes shell element. The shell is supposed to be always cut by a through crack. The element is proposed for fast transient loadings. This element is hence enriched only with jump Heavyside functions on all degree of freedom (displacements as well as rotations). The crack is discretized with segments which are more simple to implement and propagate than the level sets especially in case of large shell motions and distorsions. The mass matrix is usefull for transient analysis. For this element the mass matrix corresponding to the added DOF is simply a copy of the usual continuous DOF diagonal mass matrix. The crack propagation criterion is based on a measure of the mean stress in a small cylindrical region ahaead of crack tip. This is an extension of what has been done previously in 2D [1] and in 3D cases [3]. This mean stress state is then used te decide if the crack propagates, in which direction and at which velocity. The crack propagation criterions used here will be able to decide if it is tensile or shear crack propagation mode [2]. The element is first compared to existing 2D elements for in plane crack propagations. It is then validated by comparison of response of a fix cracked pannel under bending loading. Finally an example of simulation of crack propagation under combined membrane bending loading is proposed and compared with an SPH shell model of the same problem. A challenging experiment where one observes a crack propagation in a thin shell combined with fluid structure flow will be presented 1.

## Références

- B. Prabel, S Marie, A Combescure. Using the X-FEM method to model the dynamic propagation and arrest of cleavage cracks in ferritic steel. *Engineering Fracture Mechanics*, 75, 2984 ?3009, 2008.
- [2] ??D. Haboussa, T. Elguedj, B. Leble?, A. Combescure. Simulation of the sheartensile mode transition on dynamic crack propagations *International Journal of Fracture*, 178,195-213, 2012.
- [3] Extension de l?approche X-FEM en dynamique rapide pour la propagation tridimensionnelle de fissure dans des mate ?riaux ductiles *PHD thesis (Docinsa)*, 2013.



FIGURE 1 – A typical crack propagation path

## Fracture of Recycled Aggregate Concrete under High Loading Rates

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Demolition of old concrete structures generated a large amount of construction waste. The demolished concrete can be crushed into small particles and re-used as recycled aggregate to make new concrete, called recycled aggregate concrete (RAC). However, the mechanical properties of RAC are usually not as good as the concrete made of natural aggregate due to the residual cement paste on the surface of recycled aggregate. Many methods have been developed to improve mechanical properties of RAC. This study used a surface pretreatment method [1] to make RAC. The fracture properties of RAC under high loading rates were studied by a systematic experimental investigation as well as a theoretical modeling.

In the experimental study, the size effect method developed by Bazant was used. Geometrically similar notched-beams of RAC were used to obtain fracture properties of RAC, including fracture energy  $G_f$ , effective fracture process zone length  $c_f$ , brittleness number  $\beta$ , and R-Curves. The beams were subjected to loadings of both normal and high strain rates. In the theoretical modeling, the rate dependent mechanical properties of RAC was characterized by a theoretical model which can predict the increase of mechanical properties such as strength and stiffness with increasing strain rate based on a viscoelastic theory. The model can also predict the effect of residual cement paste and the effect of new pretreatment layer on properties of RAC based on a composite mechanics theory. The model predictions were compared with the present test data, and the results agreed quite well.

## References

 Liang Y.C., Ye Z.M., Vernerey F.J., and Xi Y. Development of processing methods to improve strength of concrete with 100% recycled coarse aggregate. Journal of Materials in Civil Engineering, 10.1061/(ASCE)MT.1943-5533.0000909, 2013.

# Impact Comminution of Solids Due to Progressive Crack Growth Driven by Kinetic Energy of High-Rate Shear

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A new theory [1, 2, 3], inspired by analogy with turbulence, was recently proposed to model the apparent dynamic overstress due to the energy that is dissipated by material comminution during penetration of missiles into concrete walls. The high-rate interface fracture comminuting the material to small particles was in that theory considered to be driven by the release of kinetic energy of high-rate shear of the forming particles and the corresponding energy dissipation rate was characterized in the damage constitutive law by additional viscosity. However, in spite of greatly improved predictions for missile impact and penetration, the calculation of viscosity involved some simplifications—one crude simplification in the calculation of viscosity from the shear strain rate, and another debatable simplification in treating the comminution as an instantaneous event, as in the classical rate-independent fracture mechanics. Presented is an improvement of the theory avoiding the former simplification. The derivation of the viscous behavior after the comminution (i.e., at post-comminution) strictly follows the principle of energy conservation, while the latter simplification is retained as admissible. The viscosity in this improved theory is inversely proportional to (7/3)-power of the strain rate, and is also proportional to its derivative, i.e. 2<sup>nd</sup> derivative of strain. This means that the comminution occurs only if the effective deviatoric strain rate changes. Another refinement of the improved theory elimin the latter simplification. The interface fracture is considered to be progressive and advance according to Evans' power law applied to the growth of interface crack area. The growth rate of interface cracks naturally leads to an additional viscosity, which allows close matching of the published test data. In both refined strategies, simplifications are avoided without making the theory significantly more complicated. In combination with the microplane damage constitutive model M7 for concrete, both refined strategies give close match of the exit velocities of missiles penetrating concrete walls of different thicknesses and of the penetration depths of missiles of different velocities into a massive block.

## Références

- [1] Z.P. Bažant, F.C. Caner. Comminution of solids caused by kinetic energy of high shear strain rate, with implications for impact, shock, and shale fracturing. *Proceedings of the National Academy of Sciences*, 110, 19291-19294, 2013.
- [2] Z.P. Bažant, F.C. Caner. Impact comminution of solids due to local kinetic energy of high shear strain rate : I. Continuum theory and turbulence analogy. *Journal of the Mechanics and Physics of Solids*, 64, 223-235, 2014.
- [3] F.C. Caner, Z.P. Bažant. Impact comminution of solids due to local kinetic energy of high shear strain rate : II. Microplane model and verification. *Journal of the Mechanics and Physics of Solids*, 64, 236-248, 2014.

## Accurate Finite Element Modeling of Stresses for Stationary Cracks Under Impact Loading

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Accurate modeling of the stress distribution in the vicinity of cracks is very important for the understanding and prediction of the crack growth and fracture of different members with cracks. In the paper we consider the numerical modeling of elastodynamics problems for stationary cracks under impact loading. One of the popular numerical approaches for such problems is based on the finite element method. However, obtaining accurate finite element solutions of elastodynamics problems under impact loading is an issue due to the appearance of large spurious high-frequency oscillations. Even time and mesh refinements can lead to divergent numerical results for impact problems; e.g., see [1, 2]. To resolve this issue, we apply the new numerical approach for elastodynamics problems that was recently developed in our papers; e.g., see [1-3] and others. It is based on the two-stage time-integration technique with the stage of basic computations and the filtering stage, on the new exact a-priori error estimator for long-term time integration, on the new finite elements with reduced dispersion. The new approach quantifies and filters the spurious oscillations, yields convergent numerical results for impact problems at the time and mesh refinements as well as it significantly reduces the computation time compared with the standard finiteelement techniques. In the paper we show that the standard finite-element techniques yield inaccurate solutions of the stress distribution for the several benchmark problems with stationary cracks under impact loading. This also leads to inaccurate calculations of the dynamic stress intensity factors. The new approach yield accurate numerical results for these problems.

- [1] A. Idesman, D. Pham. Finite element modeling of linear elastodynamics problems with explicit time-integration methods and linear elements with the reduced dispersion error. Computer Methods in Applied Mechanics and Engineering, 271, 86-108, 2014.
- [2] A. Idesman. Accurate time integration of linear elastodynamics problems. Computer Modeling in Engineering and Sciences, 71 (2), 111-148, 2011.
- [3] Idesman. A new exact, closed-form a-priori global error estimator for second- and higher-order time integration methods for linear elastodynamics. International Journal For Numerical Methods In Engineering, 88, 1066–1084, 2011.

# Gradient Damage Models and Their Use to Approximate Dynamic Brittle Fracture

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We present a gradient-enhanced continuum damage model (CDM) that can be viewed as a regularization of the variational approach to fracture (see [1] and references therein) capable of predicting the onset and space-time dynamic crack propagation in quasi-brittle materials. Similar to [1], the dynamic model will also be stated and solved in a unique variational setting involving the elastic  $\mathcal{E}$ , kinetic  $\mathcal{K}$  and Griffith surface  $\mathcal{S}$  energies

$$\mathcal{E}(\mathbf{u},\alpha) = \frac{1}{2} \int_{\Omega} a(\alpha) \mathsf{A}\varepsilon(\mathbf{u}) \cdot \varepsilon(\mathbf{u}), \quad \mathcal{K}(\dot{\mathbf{u}}) = \frac{1}{2} \int_{\Omega} \rho \dot{\mathbf{u}} \cdot \dot{\mathbf{u}} \quad \text{and} \quad \mathcal{S}(\alpha) = \int_{\Omega} w(\alpha) + w_1 \ell^2 \nabla \alpha \cdot \nabla \alpha$$

with  $w_1$  a normalization factor to recover fracture toughness  $G_c$  and  $a(\alpha) = (1-\alpha)^2$  along with  $w(\alpha) = w_1 \alpha$  two constitutive laws for damage. The scalar field  $0 \le \alpha \le 1$  depicts a continuous transition between the undamaged part  $\alpha = 0$  and the crack  $\alpha = 1$ , see Fig. 1. The damage band width is controlled by an internal length  $\ell$  related to the critical stress. The presence of the damage gradient renders the model non-local and can be seen as a regularization of classical CDM to avoid spurious mesh dependence.



Figure 1: The discrete crack  $\Gamma \subset \Omega$  approximated by a continuous damage field  $0 \le \alpha \le 1$ .

The coupled two-field continuous-time problem can then be formulated as:

- 1. Irreversibility: the damage  $\alpha(\cdot, \mathbf{x})$  is non-decreasing to prevent crack healing.
- 2. First-order stability: with the introduction of the generalized *action* integral  $\mathcal{L}(\mathbf{u}, \alpha) = \int_{I} \mathcal{E}(\mathbf{u}_{t}, \alpha_{t}) + \mathcal{S}(\alpha_{t}) \mathcal{K}(\dot{\mathbf{u}}_{t}) \mathcal{W}(\mathbf{u}_{t}) dt$ , the following variational inequality

$$\mathcal{L}'(\mathbf{u},\alpha)(\mathbf{v}-\mathbf{u},\beta-\alpha) \ge 0 \text{ for all } \mathbf{v} \in C(\mathbf{u}) \text{ and } \beta \in \mathcal{D}(\alpha)$$
(1)

leads to the classical wave equation and the crack minimality criterion

$$\begin{cases} \rho \ddot{\mathbf{u}}_t = \operatorname{div}(a(\alpha_t) \mathsf{A}\varepsilon(\mathbf{u}_t)) + \mathbf{f}_t \tag{2a} \end{cases}$$

$$\left(\mathcal{E}(\mathbf{u}_t, \alpha_t) + \mathcal{S}(\alpha_t) \le \mathcal{E}(\mathbf{u}_t, \beta) + \mathcal{S}(\beta) \text{ for all } 1 \ge \beta \ge \alpha_t \ge 0. \right)$$
(2b)

3. Energy balance: the rate of the total energy should be equal to the external power.

We use linear finite elements to discretize in space **u** and  $\alpha$ . A staggered timemarching is adopted [2] to update these two fields *separately* at every time step. The  $\beta$ -Newmark scheme is chosen for time discretization of the wave equation (2a). At **u** fixed, the convex minimization problem (2b) with bound constraints due to irreversibility will be solved by the gradient projection conjugate gradient method (GPCG) in PETSc.

In Fig. 2, we enforce the straight crack path and observes an antiplane crack propagation. Analytical solutions of the 1-d dynamic film-peeling problem are also indicated and we observe good agreement between them. In this situation the crack advances according to the dynamic Griffith criterion  $G(\dot{l}) = G_c$  during the steady propagation phase.



Figure 2: Dynamic antiplane shearing.

However all the power of the model lies in the prediction of crack kinking or branching *solely* using (2b). The results of Fig. 3 are in good accordance with [3].



Figure 3: In-plane kinking and branching.

- Kim Pham, Hanen Amor, Jean-Jacques Marigo, and Corrado Maurini. Gradient damage models and their use to approximate brittle fracture. *International Journal of Damage Mechanics*, 20(4):618–652, 2011.
- [2] B. Bourdin, C. J. Larsen, and C. L. Richardson. A time-discrete model for dynamic fracture based on crack regularization. *International Journal of Fracture*, 168(2):133–143, 2011.
- [3] Jeong-Hoon Song, Hongwu Wang, and Ted Belytschko. A comparative study on finite element methods for dynamic fracture. *Computational Mechanics*, 42(2):239–250, 2008.

# Digital image analysis of ASB assisted dynamic fracture under impact loading

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Experimental observation of fracture mechanisms is a key point to understand and further describe the physical phenomena involved in structural material failure. Crack propagation under high loading rate is a complex and strongly coupled thermomechanical problem involving large deformations, high strain rates and (quasi) adiabatic conditions. The work presented aims at giving a new insight into the process of dynamic fracture of structural materials under impact loading for further model development and calibration.

Kalthoff and Winkler-type impact tests, see [1], were carried out on double notched plates consisted of a high strength steel (the protection steel ARMOX500T). Above a critical impact velocity, the plate is seen to fail prematurely under adiabatic shear banding, see e.g. [2]. The chronology of the plate failure mechanisms was observed thanks to the use of an ultra-high speed and high spatial resolution camera (1Mfps, 312x216 pixel<sup>2</sup>). Three stages were observed: first the homogeneous then weakly heterogeneous deformation of the plate, second the propagation from the notch tip of an (adiabatic) shear band (ASB) of localised deformation, and finally the propagation of a crack within the band.

The digital images were then analysed in order to describe the aforementioned mechanisms of deformation and failure from a kinematic point of view. Two types of results were then obtained: first the evolution of the notch tip (shearing/opening) displacement/velocity and ASB tip velocity, and second the displacement (and further strain and strain rate) fields of points located along lines (quasi) perpendicular to the ASB/crack propagation path (line tracking method).

- [1] Kalthoff J.F., Winkler S., Failure mode transition of high rates of shear loading. In: Chiem C.Y., Kunze H.-D., Meyer L.W., editors. Proc. of the Int. Conf. on Impact Loading and Dynamic Behavior of Materials, Germany: Deutsche Gesellschaft für Metallkunde, DGM-Verlag, Oberursel, pp.185–95, 1987.
- [2] Longère P., Dragon A., Deprince X., Numerical study of impact penetration shearing employing finite strain viscoplasticity model incorporating adiabatic shear banding, J. Eng. Mat. Tech., ASME, 131, pp.011105.1-14, 2009.

# Simulation of dynamic tensile failure of quasi-brittle materials with a rate dependent, stress-enhanced nonlocal damage model

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The development of realistic numerical tools to efficiently model the response of concrete structures subjected to close-in detonations and high velocity impacts has been one of the major quests in defense research. Under these loading conditions, quasi-brittle materials undergo a multitude of failure (damage) mechanisms. Dynamic tensile failure (e.g. spalling), characterized by a significant strength increase associated with loading rate, has revealed to be particularly challenging to represent.

Rate dependent damage models have been successfully used to simulate this phenomenon. To minimize the pathological mesh sensitivity, revealed by some of these models, a nonlocal formulation is generally considered. Nevertheless, standard nonlocal formulations fail to properly represent damage initiation and growth around discontinuities (e.g. notches, damage areas and free boundaries). These models also exhibit unrealistic damage dispersion, which is even more pronounced under dynamic loading conditions.

These inconsistencies are the consequence of using a fixed interaction domain (characteristic length) in the nonlocal formulation. In spite of limited experimental knowledge about the definition of the internal length parameter, there is now consensus that this quantity should not be constant. It is discussed in the literature that the shape and size of the interaction domain may be dependent on the stress-state, damage evolution and geometry of the problem, i.e. boundary conditions.

In this contribution, the stress-enhanced nonlocal model developed by Giry et al. [1] has been coupled to a rate dependent damage model to study dynamic tensile failure of quasi-brittle materials. The model was implemented within the framework of LS-DYNA using a fully explicit computation scheme recently developed by de Sá et al. [2]. A study is conducted in order to evaluate the advantages and limitations of the stress-enhanced nonlocal rate dependent damage model when compared to local and standard nonlocal damage models.

- [1] C. Giry, F. Dufour, J. Mazars. Stress-based nonlocal damage model. *International Journal of Solids and Structures*, 48(25-26), 3431-3443, 2011.
- [2] J.C. de Sa, F. Andrade. Theoretical and numerical issues on ductile failure prediction an overview. *Computer Methods In Materials Science*, 10(4), 279 293, 2010.

# Inverse analysis in fracture mechanics: Comparison of two numerical methods for spalling test simulations

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The contribution deals with the numerical simulation of a modified Hopkinson Bar experiment [1] which is performed to determine dynamic material properties of Ultra High Performance Concrete (UHPC). We simulate the propagation of the stress waves as well as the evolution of cracks by using a phase field approach to fracture versus a classical cohesive element technique. In the phase field approach a parameter  $\varphi(\mathbf{x},t)$  marks the state of the material and is governed by an evolution equation stated in the sense of an Allen-Cahn equation [2]:

$$\dot{\boldsymbol{\varphi}} = -M \left[ \boldsymbol{\varepsilon}^T \mathbf{C} \boldsymbol{\varepsilon} \boldsymbol{\varphi} - G_c \left( 2l_c \nabla^2 \boldsymbol{\varphi} + \frac{1-\boldsymbol{\varphi}}{2l_c} \right) \right].$$

The effect of the energy release rate in the specimen and also the crack positions are investigated. Employing the principle of conservation of energy the specific fracture energy is calculated in the sense of an inverse analysis and compared to the values determined by experimental results. A similar procedure has been performed using a cohesive element technique including a corresponding separation law for brittle materials. The results of both applied methods are presented and compared; both approaches are evaluated with a focus on the calculation of the critical fracture energy.

- [1] H. Schuler. Evaluation of an experimental method via numerical simulations. *Frac*ture Mechanics of Concrete and Concrete Structures, 607-611, 2007.
- [2] T. Dally and K. Weinberg. The phase field method as a tool for inverse analyzes in fracture mechanics. *Accepted for publication in: Continuum Mechanics and Thermo-dynamics*, 2015.

# A local phantom node approach for crack propagation and fragmentation

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This work proposes a framework for the simulation of dynamic fracture and fragmentation of engineering structures. We present an element-local phantom node method for the three-dimensional description of dynamic crack propagation in a finite element model. Unlike standard XFEM, this method is not based on enforcement of a continuous crack path, or a level set description of crack location. Crack initiation and propagation are based on material laws much in the same way as in strong discontinuity, or enhanced strain methods, and determined on an element-by-element basis. The description of the discontinuity is local to an element. Phantom nodes are nonetheless used to facilitate complete kinematic separation of failed elements. Thus, a free surface is propagated in the material at the site of element failure in the same manner as the XFEM.

A prototype of an element-local XFEM was put forth in [1], but with a least-squares notion of crack continuity. Non-continuous cracks have also been represented with partition-of-unity element with internal cohesive zones in two dimensions in [2]. The challenges of handling phantom nodes and a free surface in three dimensions without the benefits of continuity at that surface will be addressed.

We will also address various challenges in fragmentation modeling, including the need to adapt to multiple failure models and regularizations, the transition from damage to cracking, and the need to track large quantities of fracture surfaces and new topologies. Various test problems will be presented, with comparisons to standard experimental results in brittle fracture. Extensions to ductile tearing will be discussed.

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- [1] Duan et al. Element-local level set method for three-dimensional dynamic crack growth. International Journal for Numerical Methods in Engineering 80, 1520-1543, 2009.
- [2] Remmers et al. The simulation of dynamic crack propagation using the cohesive segments method. Journal of the Mechanics of Physics and Solids 56, 70-92, 2008.

## **Dynamic Fracture of Materials and Structures**

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The behaviour of materials and structures is strongly influenced by the loading rate. Compared to quasi-static loading structures loaded by impact acts in a different way. First, there is a strain-rate influence on strength, stiffness and ductility and, second, there are inertia effects due to different reasons, which influence the resistance and failure mode. Recently, an explicit finite element code has been developed for the simulation of high velocity impact and fragmentation events. In the code the rate sensitive thermo-mechanical microplane constitutive law for brittle (e.g. glass), quasibrittle (e.g. concrete) and ductile (e.g. steel) materials can be used [1][2].

In the presentation results of a series of numerical fracture simulations in different materials are presented and discussed. The influence of loading rate for brittle, quasibrittle and ductile materials is investigated on compact tension (CT) specimen. The results of numerical investigation are compared with the own experimental results and with the results from the literature. It is shown that the progressive increase of resistance (apparent strength), which is observed in experiments for almost all materials, cannot be attributed to the increase of material strength. It is due to the inertia effects and is closely related to the material non-linearity. The progressive increase in resistance should come automatically out as a result of dynamic analysis. In brittle materials this effect is much less pronounced than in quasi-brittle (concrete) and ductile materials (steel). It is observed that maximum crack velocity, at which crack branching initiates, is not much different for ductile and quasi-brittle materials, i.e. it is in the range of approximately 500 m/s (steel) to 700 m/s (concrete). It is concluded that rate sensitivity of quasi-brittle materials is relatively strong and cannot be neglected, for instance in impact events. Finally, in case of ductile materials the size of the plastic and localization zone strongly depends on strain rate, i.e. increase in strain rate cause decrease of both. Therefore the brittleness of ductile materials increases with increase of strain rate.

- J. Ožbolt, J. Bošnjak and E. Sola. Dynamic fracture of concrete compact tension specimen: Experimental and numerical study. Int J Solids Struct, 50:4270-8, 2013.
- [2] J. Ožbolt, A. Sharma, B. Irhan and E. Sola, Tensile behavior of concrete under high loading rates. Int J Impact Eng, 69:55-68, 2014.

# Multiscale simulation of damage and fracture of ceramic nanocomposites under intensive pulse loading

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In the present work, multilevel computer simulation method [1-2] has been used for investigation of damage accumulation and fracture of ceramic composites and nanocomposites under dynamic loading. The computational models of the structured representative volume of ceramic composites were developed for materials that can be created by high temperature pressing and additive manufacturing.

Results of numerical simulation have shown that isolated micro- and meso-scale cracks can be nucleate in ceramic composites under stress pulse amplitude less than the Hugoniot elastic limit. The critical fracture stress on meso-scale level depends not only on relative volumes of voids and particles concentration, but also sizes of corresponding structure elements or skeleton sub-structure. Nucleation of local damage, activation of localized-deformation bands and self-organization process of micro-damages dispersed in SRVE are observed in the ceramic structure at high strain rates.

Results of simulation have shown the Hugoniot elastic limit and ceramics damage kinetics under dynamic loading depends on a volume concentration of nano-particles and nano-voids clusters. Self-organization process of micro-damages and occurrence of mesoscale shear band were observed in the ceramic nano-composite under compression at high strain rates. The spall stress of ceramic nanocomposites depends on relative volumes and sizes of voids and inclusions.

- [1] E. G. Skripnyak, V. V. Skripnyak, I. K. Vaganova, and V.A. Skripnyak, Fracture of Ceramic Materials under Dynamic Loadings. Proc. 19th European Conf. on Fracture (ECF19) Kazan, Russia. 639 (2012).
- [2] E. G. Skripnyak, V.A. Skripnyak, and V. V. Skripnyak, Fracture of nanoceramics with porous structure at shock wave loadings. Shock Compression of Condensed Matter. AIP Conf. Proc. 1426: 2012.

# Mass and velocity of fragments in impact fragmentation

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The most important characteristics of the fragmentation of heterogeneous solids is that the mass (size) distribution of pieces is described by a power law functional form [1, 2, 3, 4]. The exponent of the distribution displays a high degree of universality depending mainly on the dimensionality and on the brittle-ductile mechanical response of the system. Recently, experiments and computer simulations have reported an energy dependence of the exponent increasing with the imparted energy. These novel findings question the phase transition picture of fragmentation phenomena, and have also practical importance for industrial applications [5, 6]. Significantly less is known about the velocity distribution and the correlation of the mass and velocity of fragments.

We study the impact fragmentation of two- and three-dimensional disordered solids focusing on the mass distribution, spatial arrangement, and mass-velocity correlation of fragments. We developed a discrete element model of heterogeneous materials where the disordered microstructure is represented by a packing of spherical particles with a random size distribution. Cohesion is introduced such that a Delaunay tetrahedrization is performed in 3D with the particle positions and we connect the particles by elastic beams along the edges of tetraeders. The beam elements break when they get overstressed during the time evolution of the system [4, 7].

We investigate the fragmentation of plate-like objects induced by impact of a projectile in the framework of our DEM. Based on large scale computer simulations we uncover a robust mechanism which leads to the emergence of energy dependence in fragmentation processes resolving controversial issues on the problem: studying the impact induced breakup of plate-like objects with varying thickness in three dimensions we show that energy dependence occurs when a lower dimensional fragmenting object is embedded into a higher dimensional space. The reason is an underlying transition between two distinct fragmentation mechanisms controlled by the impact velocity at low plate thicknesses, while it is hindered for three-dimensional bulk systems. For thin plates at low velocities the crack structure is determined by the interference of elastic waves resulting in an essentially two-dimensional crack pattern with a regular structure. High velocity impact gradually excites cracking in the 3D bulk of the solid giving rise to a highly disordered crack structure and a steeper decay of the mass distribution. In 3D bulk samples the transition is hindered so that a unique exponent emerges. In spite of the observed non-universality of the complete mass distribution, identifying subsets of fragments dominated by different cracking mechanisms an astonishing universality of their mass distributions is revealed at all plate thicknesses [7].

A novel relation of the mass and velocity of fragments is revealed: In the damage phase fragment mass and velocity are strongly correlated. In the fragmented regime for small fragments the velocity proved to be independent of the mass, however, in the limit of large fragments a strong mass-velocity correlation occurs. The correlation function decays as a power law with a universal exponent in an excellent agreement with recent experimental findings. Our results can have relevance for industrial applications and for the understanding of the formation of meteoroid clouds and planetesimal accretion in the solar system [7].

- [1] F. Kun and H.J. Herrmann, Transition from damage to fragmentation in collision of solids, *Physical Review E* 59, 2623, 1999.
- [2] F.K. Wittel, F. Kun, H.J. Herrmann, and B.-H. Kroplin, Fragmentation of shells, *Physical Review Letters*, 93, 035504, 2004.
- [3] H.A. Carmona, F. Wittel, F. Kun, and H.J. Herrmann, Fragmentation processes in impact of spheres, *Physical Review E*, 77, 051302, 2008.
- [4] G. Timár, F. Kun, H. A. Carmona, and H. J. Herrmann, Scaling laws for impact fragmentation of spherical solids, *Physical Review E* 86, 016113 (2012).
- [5] T. Kadono and M. Arakawa, Fragmentation of brittle plates by localized impact, *Physical Review E*, 65, 035107, 2002.
- [6] N. Sator, S. Mechkov, and F. Sausset, Generic behaviours in impact fragmentation, *Europhysics Letters*, 81, 44002, 2008.
- [7] G. Pál, I. Varga and F. Kun, Emergence of energy dependence in the fragmentation of heterogeneous materials, *Physical Review E* 90, 062811, 2014.

# Addressing grid sensitivity in Peridynamics: an adaptive refinement approach

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Peridynamics [1] is a new continuum based non-local theory that has recently shown to be suitable for handling complex phenomena such as dynamic fracture and damage of both homogeneous and heterogeneous materials. Peridynamics offers many advantages over other theories since the equation of motion is stated by an integral operator and not by the differential operator used in the classical theory of mechanics. Peridynamics based software does not use ad hoc criteria in order to decide where the crack propagation will take place or to trace its direction of propagation. Moreover, it can manage both nucleation and interaction of multiple cracks simply through the solution of the equation of motion.

The most commonly used technique for the numerical implementation of the peridynamic theory is based on the use of a uniform structured grid [2]. In this way the use of the computational resources is not efficient. In a recent work by the authors [3], the adaptive refinement has been employed to increase the density of the grid only in the regions of potential damage and fracture to improve the efficiency of the numerical solution. An important parameter for the numerical implementation of peridynamics is the dimensionless ratio m, it indicates the ratio between the size of non-local interaction, so called horizon, and the grid spacing. The existing literature suggests to adopt values of m as 3 or 4 as a good compromise between accuracy and computational efficiency.

In this study we show that choosing small values of m in a uniform structured grid leads unavoidably to a dependence on the grid of the crack propagation. In order to address this deficiency, the adaptive refinement is applied to automatically increase the value of m near the crack tip, by keeping a low value of m convenient for computational efficiency far from the crack tip. A few examples about brittle dynamic fracture in 2D are presented in order to demonstrate the suitability and effectiveness of this approach.

- [1] S. Silling. Reformulation of elasticity theory for discontinuities and long-range forces. Journal of the Mechanics and Physics of Solids, 48, 175-209, 2000.
- [2] S. Silling and E. Askari. A meshfree method based on the peridynamic model of solid mechanics. Computers and Structures, 83, 1526-1535, 2005.
[3] D. Dipasquale, M. Zaccariotto, U. Galvanetto. Crack propagation with adaptive grid refinement in 2D peridynamics. International Journal of Fracture, 190(1), 1-22, 2014.

# Numerical Analyses of Fracture and Failure of Concretes under Dynamic Loading Conditions

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The cracking due to unpredicted events in concrete often creates great threat to the safety of the public construction and infrastructure of the society. It is crucial to be able to analyze the safety issue of the concrete structure. While conventional fracture mechanics based methodology is successful in many cases in evaluating effect of cracking on the safety of a concrete structure, advanced methods with more general features are needed to deal with the cracking and damage issues especially when the events of dynamic loading and damaging mechanisms are presented. In recent decades, there is a rapid development in the damage mechanics based numerical solutions. It has be implemented into many commercial finite element codes. The damage mechanics based solutions are often more general in nature in solving the damaging and cracking problems in many materials.

The concrete and many earth materials behaves often rather differently than most of the other construction materials in that its plasticity is highly dependent on the stress state. The strength differs significantly from the tensile stress state to the compression stress state, and the density changes under the applied load due to dilation as well as compaction of the porous content of the material. While these issues are rather cumbersome to solve based on the classical fracture mechanics method, it is realistic today to account for these aspects according to the damage mechanics method, especially when the computational hardware and software have been evolved rapidly in the recent decades.

Together with experimental investigations, the numerical and material models have been studied in this work. The advanced RHT (ref.[1]) and CSCM (continue surface cap method, ref[2]) model have been investigated for their capability to deal with the cracking and damage evaluation issues in concretes subjecting to high speed impact, projectile penetration, as well as the dynamic load due to the coupled blast and fragment impact.

It has been demonstrated that by considering the non-homogeneous plasticity, the strength as a function of all the three stress invariants, as well as the fracture energy and strength as a function of the strain rate, reasonable estimation of the cracking and damaging process in the concrete material may be achieved even though at the high strain rate, the material model and constitutive equation still need improvement.

The commercial hydrocode Ls-Dyna (ref.[3]) and Autodyne are used in this study. Their advantages and shortcomings are compared. For the dynamical loading involving the coupled action of both the fragment penetration and the blast loading, it has been shown that the computational efficiency should be investigated due to the nature that the analyses have to be performed to cover both the high speed penetration of the fragment and the relative slow process of the blasting load.

### References

[1] W. Riedel: Beton unter dynamishen lasten, Meso- und makromechanische modelle und ihre parameter. EMI-Bericht 6/00, Freiburg, July 2000.

[2] Y. D. Murray: Users manual for Ls-Dyna concrete material model 159, APTEK, Inc., FHWA-HRT-05-062, Contract: DTFH61-01-C-00075, May 2007

3. LS-DYNA®, KEYWORD USER'S MANUAL, VOLUME II, Material Models, LS-DYNA R7.1, May 19, 2014 (revision: 5442)

# Peridynamic modeling of crack branching and impact on polycrystalline ceramics

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Crack branching in brittle materials happens for two main reasons: in homogeneous and isotropic materials, like glass, crack branching is induced by the dynamics of the crack propagation ([1], [2]); in heterogeneous materials, inclusions and weak interfaces can force a crack to go around it, thus splitting the crack even under quasi-static crack growth conditions.the dynamic effects involved in crack branching in isotropic and homogeneous materials lead to a limitation of the crack propagation speed, which, in most experimental observations, rarely goes above  $0.6C_R$ , where  $C_R$  is the Rayleigh wave speed of the material. The fact that crack branching is not, however, controlled by the crack propagation speed has been discussed in the experimental literature for some years. In this talk I will present results obtained with a peridynamic model that explain the crack branching phenomenon in a new light. We show that, indeed, the evolution/flow of strain energy around the crack tip controls crack branching, and that crack may branch, in isotropic materials, even when the crack runs at  $0.3-0.4C_R$ .

With the confidence gained in capturing experimentally observed characteristics of dynamic brittle fracture, we extend a peridynamic model to polycrystalline ceramic materials. We simulate a 3D polycrystalline AION sample of mm<sup>3</sup> dimensions (with about 200 grains generated by Voronoi tessellation) under edge-on impact conditions to investigate the mechanisms of dynamic brittle fracture. In the peridynamic model, we use a nonlocal region (the PD horizon) that is 1/10 of the average grain size. This is small enough to simulate the influence of microstructure on failure front and fracture propagation. Simulation results match experimental results very well. The peridynamic results include mixed intergranular and transgranular fracture, observed in AION fracture ([3]). The experiments notice a coherent damage zone moving through the material at relatively high speed. When the failure front stops from advancing, distributed damage transitions to individual, localized cracks that initiate from the edge of the failure front and move at a much lower propagation speeds that the failure front. Both the failure mechanisms and the propagation speeds are well captured by our PD model.

- [1] Y.D. Ha, F. Bobaru. Characteristics of dynamic brittle fracture captured with peridynamics. *Engineering Fracture Mechanics*, 78(6), 1156-1168, 2011.
- [2] F. Bobaru, G. Zhang. Crack branching in dynamic brittle fracture: peridynamic results. to be submitted, 2015.
- [3] G.F. Zhang, Y. Wang, F. Bobaru. Peridynamic modeling of dynamic brittle failure in polycrystalline ceramics. in preparation, 2015.

- chapter 6 -



# MS6: Fracture and contact, fretting, cohesive interface models



# A 3D multiscale cohesive zone model accounting for friction, damage and interlocking

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Cohesive zone models (CZMs) are widely used to simulate the behaviour of structural interfaces in quasi-brittle materials along surfaces where formation and propagation of cracks is expected. They represent an effective alternative approach to fracture mechanics based methods for a wide variety of problems at very different scales, such as crack growth in dams, mortar-joint failure in brick masonry, bond slip response of reinforcing bars in concrete, debonding of adhesive joints, delamination or fibre matrix, debonding in composites, among many others. With increasing mode II/mode I ratio an increasing amount of energy is dissipated by friction. Therefore an accurate mechanical description of these damage behaviours has to account for dissipation induced by fracture and friction and for the role played by the geometry of the asperities along the interface with the associated interlocking effect.

Several interface models accounting for damage-friction coupling have been proposed in literature, see, e.g., Del Piero and Raous [1] and references therein. Some of them are based on nonassociative softening plasticity, such as the multidissipative interface model proposed by Cocchetti et al. [2] and the contributions given by Bolzon and Cocchetti [3] and by Cervenka et al. [4] in the field of concrete dams analysis, and by Giambanco et al. [5]. A cohesive approach with separate values of the energy dissipation in mode I and mode II combined with a contact algorithm has been also recently investigated by Snozzi and Molinari [6].

In [7] a model based on a simplified micromechanical formulation has been recently proposed whose main idea is to describe the asperities of the interface in the form of a periodic arrangement of distinct inclined planes, denominated Representative Interface Element (RIE), the interaction within each of these surfaces being governed by the

interface formulation proposed in [8]. Specific features of the model are the following: 1) each microsurface is assumed to be decomposed into an undamaged and a fully damaged part; 2) the evolution of damage is assumed to depend on the elastic energy in the undamaged part; 3) it is assumed that friction occurs only on the damaged part and is governed by a Coulomb law.

In this contribution an extension of the formulation [7] to 3D problems is presented whereby the RIE is represented as a 3D pattern of a number of inclined surfaces and the formulation allows defining relative displacements of the microsurfaces along three possible independent directions. It is shown that, although a non-isotropic RIE composed of 5 inclined planes is employed, a quasi-isotropic behaviour is retrieved for in-plane tangential decohesion paths. Consistency with the response provided by the 2D-model [7] is also shown.

- [1] G. Del Piero and M. Raous, A unified model for adhesive interfaces with damage, viscosity and friction. *Eur J. Mech. A/Solids*. Vol. **29**, pp. 496-507, 2010.
- [2] G. Cocchetti, G. Maier and X.P. Shen. Piecewise linear models for interfaces and mixed mode cohesive cracks. *Comp Model. Eng. Sci.* Vol. **3**, pp. 279–298, 2002.
- [3] G. Bolzon, G. Cocchetti, Direct assessment of structural resistance against pressurized fracture. *Int. J. Num. Anal. Meth. Geomech.*, Vol. 27, pp. 353–378, 2003.
- [4] J. Červenka, J.M. Chandra Kishen, V.E. Saouma. Mixed mode fracture of cementitious bimaterial interfaces; part II: numerical simulation. *Eng. Frac. Mech.* Vol. 60, pp. 95–107, 1998.
- [5] G. Giambanco, S. Rizzo, R.. Spallino Numerical analysis of masonry structures via interface models. *Comp. Meth. Appl. Mech. Eng.*, Vol. **190**, pp. 6493–6511, 2001.
- [6] L. Snozzi, J. F. Molinari, A cohesive model for mixed mode loading with frictional contact capability. Int. J. *Numer.*. *Meth. Engng.* Vol. **93**, pp. 510-526, 2012.
- [7] E. Sacco, R. Serpieri and G. Alfano, A thermodinamically consistent derivation of a frictional-damage cohesive-zone model with different mode I and mode II fracture energies, *Eur J. Mech. A/Solids.*, Vol **49**, pp. 13-25, 2015.
- [8] G. Alfano, E. Sacco, Combining interface damage and friction in a cohesive-zone model. *Int. J. Num. Meth. Eng.* Vol. 68, pp. 542-582, 2006.

# A quantitative investigation of the competition between cohesive and adhesive fracture at interfaces

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In many fracture problems at interfaces between dissimilar materials, the crack deflects from the actual interface into the neighbouring bulk material. A good example is crack deflection that is observed in roughened polymer-copper interfaces [1]. Depending on the nature of the roughness and the amplitude of the asperities, an interface crack can kink into the bulk polymer material and propagate there, dissipating a significant amount of energy. It is strongly believed that this phenomenon is one of the main causes for the increased strength and toughness of roughened or patterned interfaces [1].

In order to quantify the material and geometric parameters that cause this kinking behaviour, numerical models can be of assistance. One of the methods that can be used is the cohesive zone method. The polymer-copper interface is modelled by means of interface elements with a given traction separation law [2]. Crack growth away from the interface into the polymer bulk material is modelled with the cohesive zone method as well. In order to facilitate crack nucleation and subsequent growth in arbitrary positions and directions in the polymer, interface elements are placed between all bulk continuum elements. These interfaces are initially rigid and activated by means of a failure law [3]. Once active, the opening of these elements is governed by a traction separation law.

The model has been used successfully to simulate crack deflections at roughened surfaces, at least in a qualitative fashion, see Figure 1. One of the conclusions of this analysis was that the crack deflection is driven by the ratio of fracture strengths of the interface and the bulk materials and not so much by the difference in fracture toughnesses. This is in contradiction with similar studies, which mainly focus on the kinking behaviour in relation to differences in toughnesses, e.g. [4, 5].

In this presentation, we will further examine the crack kinking problem and provide a quantitative analysis, developing failure maps in which both the ratios in strength and toughness of the interface and bulk cohesive zone model are taking into account, as well as the dummy stiffness and propagation criterion.



Figure 1: (a) Adhesive and cohesive cracking after loading of a patterned polymer-copper interface. (b) Finite element mesh. (c) Simulated crack paths. The colours of indicate the amount of damage in the cohesive zone. Blue is undamaged, red is completely damaged [1].

- [1] O. van der Sluis, J.J.C. Remmers, M.A.C. Thurlings, B.J. Welling and S.P.M. Noijen. The competition between adhesive and cohesive fracture at a micro-patterned polymer-metal interface. *Key Engineering Materials*, **577**, 225–228, 2013.
- [2] X.P. Xu and A. Needleman. Numerical simulations of fast crack growth in brittle solids. *Journal of the Mechanics and Physics of Solids*, **42**, 1397–1434, 1994.
- [3] G.T. Camacho and M. Ortiz. Computational modelling of impact damage in brittle materials. *International Journal of Solids and Structures*, **33**, 2899–2938, 1996.
- [4] M.Y. He and J.W. Hutchinson. Crack deflection at an interface between dissimilar elastic materials. *Journal of Applied Mechanics*, **111**, 270–278, 1989.
- [5] J.L. Strom and J.P. Parmigiani. Transition of crack path at bi-material interfaces. *Engineering Fracture Mechanics*, **115**, 13–21, 2014.

# Mesoscopic analysis of concrete failure behavior under high temperature effects

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Long term exposures to high temperature strongly affect most of concrete relevant mechanical features such as cohesion, friction, stiffness and strength. These severe degradations lead to irreversible damage or, moreover, sudden collapse of the related structures.

In this work, the failure behaviour of concrete subjected to high temperature effects is analysed at the mesoscopic level of observation, whereby three phases are recognised: aggregates, mortar and aggregate-mortar interfaces. While the aggregate are modelled by means of thermo-elastic formulations, the degradation of the mortar and of the joints between mortar and aggregates are modelled by means of non-linear fracture energy-based thermoplastic interfaces, in the framework of the discrete crack approach. This interface formulation, see [1], takes into account the degradation of strength and fracture energy properties, both in mode I and II, due to temperature, as well as the effect of temperature on the non-associative properties.

This contribution focuses on the analysis of finite element predictions of failure behaviour of concrete members subjected to different thermo-mechanical conditions at the mesoscopic level of observation. In particular, the variation of failure mechanism and ductility, in pre- and post-peak are evaluated, as well as the influence of the ratio aggregate/mortar in the overall temperature dependent failure behaviour.

### References

 A. Caggiano, G. Etse, Zero-thickness interface model for coupled thermomechanical failure analysis of concrete. Submitted to Computer Methods in Applied Mechanics and Engineering, 2015.

# Simulation of ductile fracture of aluminum alloy using a triaxiality dependent cohesive zone model

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There has been significant interest in extending the applicability of the cohesive zone model to a wider range of crack growth simulations by incorporating the effect of triaxiality on the material degradation in the process or cohesive zone. The conventional model parameters, cohesive energy and cohesive strength, typically used to describe the constitutive behavior of the process zone through a traction-separation law, have been shown to depend on the triaxiality of the stress state [1, 2]. In this approach, however, the effect of triaxiality on the model parameters has been first established using a plane strain unit cell subject to bi-axial stress with the damage evolution modeled by porous metal plasticity model.

More recently another approach has been proposed which retains the simplicity of cohesive zone model by explicitly incorporating the triaxiality of stress-state in their formulation using the basic mechanical properties and two model parameters C and S defining the upper and lower bounds of equivalent plastic strain at failure respectively [3]. The model has been shown to effectively reproduce experimental data from the ductile fracture testing of a wide range of geometries for a mild steel IS 2602 [4].

In the present work, the triaxiality dependent model is applied to an aluminum alloy that exhibits less ductility compared to the mild steel studied in [4]. For simulation, the triaxiality dependent model was implemented as linear displacement formulation based elements and they were placed at the likely plane of ductile fracture. The tractionseparation law of these elements was dependent on the stress-state of the neighboring continuum elements. Model parameter, C, that defines the lower bound on the equivalent plastic strain at failure is first determined from the tensile test data of a single notched bar geometry. Using this model parameter in plane strain analyses of a range of notched bar geometries, the predicted failure loads correspond well with the experimental data as shown in the form of the failure locus in Figure. 1(a). Further, the second model parameter, S, was estimated from the fracture testing and simulation of a compact test specimen with a/w = 0.54. It is shown that the macroscopic response of the experiments was best reproduced when a plane strain core of thickness 0.05 times thickness of the specimen was used. Using this combination of model parameters and plane strain core height, the model was shown to be effective in prediction of the macroscopic response for a wider range of initial crack lengths as shown in Figure. 1(b).



FIGURE 1 – (a) Locus of equivalent plastic strain at failure for notched bar geometries. (b) The macroscopic response from experiments and simulations with PSC = 0.05B, S/C = 9.

## Références

- [1] Siegmund and Brocks. Prediction of the Work of Separation and Implications to Modeling, *International Journal of Fracture*, 99, 97-116, 1999
- [2] Siegmund and Brocks. A numerical study on the correlation between the work of separation and the dissipation rate in ductile fracture, *Engineering Fracture Mechanics*, 67, 139-154, 2000
- [3] Banerjee and Manivasagam. Triaxiality dependent cohesive zone model. *Engineering Fracture Mechanics*, 76(12), 1761-1770, 2009
- [4] Rashid and Banerjee. Implementation and validation of a triaxiality dependent cohesive model : experiments and simulations. *International Journal of Fracture*, 181, 227-239, 2013

# A mesoscopic approach to study the influence of aggregates spatial arrangement on concrete dynamic behavior

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The strength variability of the concrete-like materials is usually represented, in phenomenological macroscopic models, using stochastic tools that need to be identified with a large number of experimental tests. The variability phenomenon is then usually linked to the non-uniform characteristics of the material at the mesoscopic scale, due to the various phases of the concrete. An alternative to a macroscopic model with numerous parameters is to represent these phases explicitely.

In this work, we intend to explore the influence of the heterogeneous meso-structure of the concrete (aggregates in a cement paste matrix) on its dynamic properties. For this purpose, we use FE simulations with a cohesive approach and an explicit representation of the meso-structure. A 2D geometrical model of concrete consisting of aggregates, interfacial transition zones and a matrix is used. That kind of approach has been previously been used to give a better understanding of the fracture mechanisms ([1], [2]) and gave relevant results on the Dynamic Increase Factor (DIF) experimentaly observed. However, if the scale of the numerical specimen is large enough, that means that the material can be seen as homogeneous, and we are not able to reproduce the experimental variability on the tensile strength [3]. The influence of the spatial disposition of the elements of the mesostructures is still open for discussion. Specifically, the impact of the clustering of aggregates on crack initiation and propagation. In our study, we propose to analyse the effect of such groups of aggregates later called clusters. In order to detect and mesure the clustering, we developped a statistical indicator inspired from a global estimator, Ripley's function [4]. Numerical results coupling the mesure of the clustering with its effect on the global fracture process will be presented.

#### Références

[1] Fabrice GATUINGT, Léonardo SNOZZI, and Jean-François MOLINARI. Numerical determination of the tensile response and the dissipated fracture energy of concrete : role of the mesostructure and influence of the loading rate. *International Journal for Numerical and Analytical Methods in Geomechanics*, 37(18) :3112–3130, 2013.

- [2] Joško OžBOLT, Josipa BOŠNJAK, and Emiliano SOLA. Dynamic fracture of concrete compact tension specimen : Experimental and numerical study. *International Journal of Solids and Structures*, 50(25) :4270–4278, 2013.
- [3] P FORQUIN and B ERZAR. Dynamic fragmentation process in concrete under impact and spalling tests. *International Journal of Fracture*, 163(1-2):193–215, 2010.
- [4] Brian D RIPLEY. Spatial statistics, volume 575. John Wiley & Sons, 1981.

## On the development of a cohesive model for crack propagation in wood under relative humidity variations

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The fracture mechanic approach is more and more performed to estimate time to failure of timber structures. In this approach, the cohesive zone model (CZM) has been widely employed in numerical simulations to describe the fracture analysis in various materials and structures. The cohesive zone represents the fracture process zone (FPZ) at the crack tip and its behavior can be defined in terms of a traction-separation law (STL) [1]. The damage failure of joint elements in CZM is characterized by a progressive degradation of the material stiffness in the joints. Advantages of the CZM are : i) it does not require the existence of a pre-crack, ii) the crack and the crack propagation is described without remeshing to avoid the cumbersome due to the mesh updating to match the geometry discontinuity.

Wood mechanical properties depend on the temperature and on the moisture content (MC) [2]. Varying MC induces internal stresses which may cause the crack propagation because the fracture process zone at the crack tip is directly submitted to air relative humidity (RH) variations [3]. Some authors have studied the analysis of crack growth process in the orthotropic–viscoelastic material (such as wood) under mode I by considering the progressive crack-lip decohesion [4]. This progressive field is determined by evaluating the stress perturbation caused by an instantaneous crack advance.

In this research, we focus on the influence of the moisture on the fracture process zone during the crack growth in wood. The aim of this study is to develop a computational methodology to model the crack growth under the environmental condition in the timber structure. Based on the classic cohesive zone model, a new model, introducing the influence of the moisture content on the cohesive zone, is proposed and implanted in source Esope in the finite element code Cast3m. Under the external loading and the variation of the relative humidity, the tridimensional crack growth simulation is carried out using a mechano-sorptive behaviour for the wood substrate. The mechano-sorptive approach, which was first developed by Dubois et al [5], uses a coupling between moisture variations with mechanical properties of wood.

Fracture tests in mode I on the Maritime pine are performed on a modified Tapered Double Cantilever Beam specimen (mTDCB) under different constant moisture contents in order to determine CZM parameters in which the bilinear softening traction-separation law is used. Then, the other fracture test is carried out in a climate room under the relative humidity variation combined with a constant temperature.

In our analysis, Fick's law for the mass transfer is used for the moisture diffusion problem in wood. We assume that the surface moisture content of the specimen is at equilibrium of MC in the sorption isotherm curve (RH-MC) and the surface emission coefficients are similar for three directions (Longitudinal, Tangential and Radial) in wood. When the crack propagates, the boundary of the moisture surface exchange is changed which demands a geometric remeshing and then a projection of the hygrothermal field before the crack propagation on the new geometry (with the new crack length). At the beginning of a current time step, the moisture content filed of the specimen is obtained; and then, elastic properties depending on the moisture content in the whole specimen are updated. In condition of the constant temperature, the rheological model of wood is simply composed of two deformation mechanisms which simulate the elastic response and the viscoelastic creep. In this paper, we have limited the mechano-sorptive behavior for wood substrate at an elastic behaviour.

During the time increment, due to the rapid variation of moisture, the crack opening at the cohesive interface is considered constant; the damage state of joint elements is assumed to be independent on the moisture. An additional stress increment within the cohesive zone is calculated using the influence of the moisture on the mechanical properties of the cohesive zone (such as the stiffness). At the new time step, the additional stress increment is converted into the external force increment inducing on the cohesive zone. When the crack develops, the joint element is damaged and the Traction-Separation Law (TSL) corresponding to the moisture at the tip of FPZ is used. This TSL law for this damaged zone does not change for next processes of the crack growth. These calculations have been implemented in the finite element code.

Our numerical simulation results are compared with the experimental data. By taking into account the viscoelastic mechano-sorptive behaviour for the wood substrate, our proposed computational methodology will be used to predict the time to failure of timber structure under environmental conditions.

- [1] Bazant ZP, Planas J (1998) Fracture and size effect in concrete and other quasibrittle materials. CRC Press
- [2] Guitard D., "Le bois, matériau d'ingénierie, chapitre III: Comportement mécanique du bois" (1987).
- [3] Chaplain M, Valentin G (2010) Effects of relative humidity conditions on crack propagation in timber: experiments and modelling. World Conf. Timber Eng. pp 1–8
- [4] Dubois F, Chazal C, Petit C (2002) Viscoelastic crack growth process in wood timbers : An approach by the finite element method for mode I fracture. Int J Fract 113:367–388.
- [5] Dubois F, Husson J-M, Sauvat N, Manfoumbi N (2012) Modeling of the viscoelastic mechano-sorptive behavior in wood. Mech Time-Dependent Mater 16:439–460. doi: 10.1007/s11043-012-9171-3

- chapter 7 -



# MS7: Fracture and damage of composites and laminates



# Simulation of ply damage and delamination in multilayer composites to predict the residual compressive strength after impact loading

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Compression after impact tests are a standardized and widely used method to evaluate the impact resistance of composite materials. Explicit finite element simulations are presented for modeling the mechanical response of fiber reinforced polymer composites under low velocity impact and subsequent quasi-static compression. Impact loading on multilayer composites can cause interface and ply damage inside the laminate. Delamination divides the laminate into sub-laminates which have a lower bending stiffness than the original laminate and reduces, in combination with ply damage, the compressive strength significantly. Therefore, compression after impact simulations are carried out to predict the residual compressive strength of an impact damaged multilayer composite plate.

Material related nonlinearities in laminated composites are mainly delamination between the plies and the elasto-plastic-damage ply behavior. The focus is set on the simulation of these two mechanisms and their interaction. A stacked shell approach is utilized where each ply is represented by its own layer of shell elements. The individual layers are connected by cohesive zone elements representing the interface. The damage and failure behavior of the fabric plies are modeled using continuum damage models with (i) a so called UD-analogy approach and (ii) a weave material model. In the UD-analogy approach each fabric ply is represented by two unidirectional (UD) reinforced layers with orthogonal fiber directions which are perfectly bonded and stacked in a composite shell definition. The weave material model describes the constitutive behavior of a fabric ply as homogeneous orthotropic material. The degradable interfaces are modeled by a cohesive law formulation with traction separation behavior. In case of interface failure, contact interactions between the plies are enforced.

An explicit solution scheme is employed, i.e. ABAQUS-Explicit v6.13 (*Dassault Systèmes Simulia Corp., Providence, RI, USA*), to simulate a compression after impact test procedure according to the standardized ASTM / BOEING method. Various models with different fiber orientations and impact energies are presented. The advantages and disadvantages of the modeling approaches are explained and the results discussed. Finally the obtained simulation results are compared to test results.

# Void shape effects and porosity ratcheting of elasto-plastic materials subjected to cyclic loading

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It is a well known fact that pressure plays an important role in both high and low cycle fatigue regime [1, 2, 3]. A large amount of experimental data has shown that hydrostatic stress has a strong influence of the lifetime as a function of the loading conditions. Moreover at the microscopic level of the material one can observe an evolution of damage and defects. as the macroscopic hydrostatic stress component will induce an evolution of plasticity and of the microstructure in the presence of voids or inclusions. Although significant advances have been made in the last decades in ductile fracture under monotonic loading conditions (see for instance [4, 5]), a lot of questions remain open in the context of cyclic loading.



Figure 1: Contours of the maximum principal logarithmic strain at 40 cycles in the case of stress triaxiality  $X_{\Sigma} = 3$  (u/L = 1%) for Lode angles (a)  $\theta = 0^{\circ}$  and (b)  $\theta = 30^{\circ}$  and (c)  $\theta = 60^{\circ}$ .

In this regard, this work [6] investigates the effect of cyclic loading upon periodic elasto-plastic porous materials. The aim is to understand the evolution of the underlying microstructure, described here with a single void embedded in a cubic unit-cell. Periodic finite element (FEM) calculations are carried out under a finite strain deformation process keeping the absolute value of the stress triaxiality and the Lode angle constant during the cycle. As a result of the applied loading conditions, the void geometry, both volume and shape, change significantly leading to porosity and void shape ratcheting. The void shape becomes non-spherical from the very first cycle leading to a markedly asymmetric

cyclic response of the material. This, in turn, results in an observed maximum stress as a function of the number of cycles. In addition, even though the average applied strains are relatively small, the local strains near the void surface increase significantly inducing a significant localization of the deformation. Finally, several initial void shape configurations are also considered. In the majority of the cases studied, the void evolves into a crack-type shape in the direction of the minimum absolute stress. This, in turn, is consistent with a configuration corresponding to a crack subjected to a mode I cyclic loading.

- Constantinescu, A., Dang Van, K., Maitournam, M. H., A unified approach for high and low cycle fatigue based on shakedown concepts, *Fatigue Fract. Eng Mater. Struct.* 26, 561-568, 2003
- [2] Monchiet, V., Charkaluk, E., Kondo, D., A micromechanical explanation of the mean stress effect in high cycle fatigue, *Mechanics Research Communications* 35, 383-391, 2008.
- [3] Morel, F., Bastard M., A multiaxial life prediction method applied to a sequence of non similar loading in high cycle fatigue, *International Journal of Fatigue* 25, 1007-1012, 2003.
- [4] Devaux, J., Gologanu, M., Leblond, J.B., Perrin, G. On continued voids growth in ductile metal subjected to cyclic loadings, *Symposium on Nonlinear Analysis of Frac-ture, J.R. Willis ed.*, 299-310, 1997.
- [5] Danas K., Aravas N., Numerical modeling of elasto-plastic porous materials with void shape effects at finite deformations, *Composites : Part B* 43, 2544-2559, 2012.
- [6] M. Mbiakop, A. Constantinescu, K. Danas, On void shape effects of periodic elasto-plastic materials subjected to cyclic loading, *European Journal of Mechanics A/Solids*, 49, 481-499, 2015.

# Numerical Simulation and Analysis of a Progressive Failure of Notched Composite Laminates Based on Elastoplastic Damage Model

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A finite-element model which includes in-ply and delamination damage effects is developed for progressive failure analyses of notched laminates composed of advanced composite materials. The combined elastoplastic damage model is employed to represent the in-plane mechanical response of the composite layers [1]. A cohesive zone model based on cohesive elements available in Abaqus is implemented to simulate delamination behaviour in the adhesive interfaces [2]. Continuum shell elements and cohesive elements are stacked together to simulate composite and adhesive layers, respectively.

The strain-driven implicit integration algorithm is developed based on equations of continuum damage mechanics, plasticity theory and using return mapping procedure. To ensure the algorithmic efficiency of the Newton-Raphson method in the finite element analysis, a tangent operator that is consistent with the integration algorithm is derived. The model is implemented in Abaqus using a user-defined subroutine (UMAT).

The progressive failure of a GLARE specimen under tensile loading is simulated using the aforementioned modelling approach. The specimen is made of four aluminium 2024-T3 alloy layers alternating with three S2-glass/FM94-epoxycomposite laminates, and contains the central hole. Comparison of the computational and experimental results confirms the validity of the finite-element analysis.

An investigation into effects of the layup configurations (i.e., unidirectional, cross-ply and quasi-isotropic) and the numbers of holes on the mechanical behaviour of perforated aluminium/CFRP (carbon-fibre reinforced plastic) laminates subjected to tensile loading has been undertaken. The stress distribution and damage progression modes together with the failure sequences are presented for various laminate configurations having different perforation patterns [3].

Results of the progressive failure analysis of double-notched AS4/3501-6 carbon-epoxy laminate subjected to in-plane tensile loading are discussed. The predicted load-displacement curve is compared with those obtained from the experiments and

predicted by other material models.

It has been shown that the numerical model proposed in this study provides an efficient simulation methodology for the progressive failure analysis of notched laminated composite structural components made of materials exhibiting both the plasticity effects and susceptibility to the delamination.

- [1] J.F. Chen, E.V. Morozov and K. Shankar. A combined elastoplastic damage model for progressive failure analysis of composite materials and structures. Composite Structures, 94(12), 3478-3489, 2012.
- [2] J.F. Chen, E.V. Morozov and K. Shankar. Simulating progressive failure of composite laminates including in-ply and delamination damage effects. Composites Part A, 61, 185-200, 2014.
- [3] J.F. Chen, E.V. Morozov and K. Shankar. Progressive failure analysis of perforated aluminium/CFRP fibre metal laminates using a combined elastoplastic damage model and including delamination effects. Composite Structures, 114, 64-79, 2014.

# A general Finite Fracture Mechanics model for crack initiation in adhesive lap joints

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Due to their various advantages over traditional assemblies adhesive joints are increasingly used in many industrial applications. For a reliable use of adhesive joints in structural engineering a solid understanding of the physical failure processes are necessary. Moreover, efficient and robust analysis methods that allow for a precise prediction of the failure loads are crucial, especially in early design stages such as pre-dimensioning or in optimization processes.

Former studies have shown that the coupled stress and energy criterion proposed by Leguillon [1] settled in the framework of Finite Fracture Mechanics is an encouraging approach for the modeling of crack initiation in bonded lap joints [2, 3]. However, the current failure models for adhesive joints in literature are joint specific and mainly focus on failure load predictions of single and double lap joints.

In the present work, a general sandwich-type model for the stress analysis of bonded joints with shear flexible composite adherends by Weißgraeber et.al. [4] is presented for the application to various joint designs. This model is combined with the coupled criterion to derive a general failure model that is applicable to various joint configurations, as e.g. single lap joint, double lap joint, L-joint, T-joint and reinforcement patches. To allow for a quick determination of the failure load the corresponding optimization problem is solved by a highly efficient iterative solution scheme.

Additionally, a numerical approach by using Cohesive Zone Models in a Finite Element analysis is implemented to predict crack initiation in adhesively bonded lap joints. This widespread and well-established method is used as a reference solution to the proposed approach.

In a comprehensive study a comparison of the results for exemplary joint designs to experimental test series from literature is performed. Moreover, the influence of geometrical parameters on the failure load is discussed in detail for specific joint designs. It is shown that a good agreement is obtained for all observed configurations.

#### References

[1] D. Leguillon. Strength or toughness? A criterion for crack onset at a notch. *European Journal of Mechanics - A/Solids*, 21(1), 61-72, 2002.

- [2] P. Cornetti, V. Mantič, A. Carpinteri. Finite fracture mechanics at elastic interfaces. *International Journal of Solids and Structures*, 49(7-8), 1022-1032, 2012.
- [3] P. Weißgraeber, W. Becker. Finite Fracture Mechanics model for mixed mode fracture in adhesive joints. *International Journal of Solids and Structures*, 50, 2383-2394, 2013.
- [4] P. Weißgraeber, N. Stein, W. Becker. A general sandwich-type model for adhesive joints with composite adherends. *International Journal of Adhesion & Adhesives*, 55, 56-63, 2014.

# Pattern based description of a CMC yarn failure combining GFEM multi-scale & Finite Fracture Mechanics

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In this work a GFEM multi-scale method for modeling failure of the CMC yarn under tension is presented. Using a pattern-based description of the microscale [1], the construction of the model falls into two main parts : the modeling of the CMC yarn as a multiphase composite, and the modeling of crack events inside the microstructure. The classical difficulties associated with the implementation and computation cost of the GFEM [2] are dealt through a multiscale approach based on the Saint Venant principle. This multiscale vision enables the microstructure and the microkinematics to be handled on the scale of the pattern regardless of the discretization on the macroscale. The multi-scale approach is implemented inside the finite element software Cast3M in order to use the existing infrastructure of the open source code for the GFEM integration.

Here the objective is to extract the most important information from the microscale using a specific number of patterns (handbook problems) that are neccesary to reproduce the complex behavior of CMCs as accurate as possible. Due to the fact that an enormous amount of information is inherent in the microscale, this task is very important for providing an insight into the dominating mechanisms. A library of numericaly generated enrichment functions is created by solving local boundary value problems (handbooks) for the most relevant patterns. The solutions of these problems can then be used in the GFEM by superimposing the handbook meshes i.e. the meshes used in the numerical construction of the handbook functions over the GFEM mesh after appropriate translation and rotation. The computation of local problems can be parallelized without difficulty allowing the solution of very difficult problems.

A direct extension for the simulation of crack events is incorporated into the model inside the framework of finite fracture mechanics for treating arbitrary discontinuities without remeshing. This comes from the fact that many failure processes of CMCs i.e. fiber breaks, matrix cracks and fiber/matrix debonding are characterized by fracture events instead of by continuous crack growth. Finite fracture mechanics is able to control the growth of such defects [3]. It is assumed that a crack begins to propagate when the finite energy release rate  $\Delta G$  equals a critical energy release rate  $\Delta G_c$  for the specific event. This step by step crack formation appear instantaneously on the experimental time scale and is not possible or of interest to follow the history of its development.



FIGURE 1 – Left : Crack path between inclusions (no fiber/matrix debonding is considered); Middle : Handbook problem with a single inclusion; Right : Superimposed handbook pattern of a matrix crack connecting two handbook problems (rotated).

A considerable part of the present work is concerned with the mechanics of crack initiation and propagation studing numericaly offline quite a lot of pattern behaviours. Specificaly, the competition between interface crack extension and kinking (assuming Mode I & II propagation after kink) is formulated on the energetic basis comparing ratios of the corresponding energy release rate for a load level ( $\Delta G^{int}$  and  $\Delta G^{kink}$ ), respectively, associated to the interface and the incipient kinked crack [4]. Adddtionally, specific criteria have to be considered when a propagating crack is reaching a fiber/matrix interface. It has to be noticed that due to the multi-scale description the computation of the energy release rate at the handbook level is very efficient regarding the overal computational cost.

The final aim is the application of the method for a large number of inclusions in order to investigate what statistically in the microstucture leads to damage, or in other words, how the local microstructure interacts and cause damage.

#### Références

- Tranquart, B., Ladevèze, P., Baranger, E., Mouret, A., A computational approach for handling complex composite microstructures, *Composite Structures*, 94(6), 2097-2109, 2012.
- [2] Strouboulis T., Babuska I., Copps K., The design and analysis of the generalized finite element method, *Computer Methods in Applied Mechanics and Engineering*, 181(1-3), 43-69, 2000.
- [3] Hashin Z., Finite thermoelastic fracture criterion with application to laminate cracking analysis, *Journal of the Mechanics and Physics of Solids*, 44(7), 1129-1145, 1996.
- [4] He M.Y., Hutchinson J.W., Kinking of a crack out of an interface, *Journal of Applied Mechanics*, 56(2), 270-278, 1989.

# Transverse cracking scenarios under oxidation and mechanical loading in polymer matrix composites

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At relatively high temperatures ( $150-300^{\circ}$  C), polymer oxidation leads to differential shrinkage and evolution of the properties of polymer matrix composites. These local modifications of a thin surface layer may lead to oxidation-related cracking and they generally modify the 'classical' cracking scenario during mechanical loading.

In this work,  $[90/0]_s$  specimens subjected to oxidation and mechanical loading are simulated via Finite Fracture Mechanics (FFM). The effectiveness of FFM for the prediction of transverse cracking has been previously demonstrated [1, 2]. Here, we use it to account for the competition between cracking scenarios.

A finite element model including the oxidized layer was used to compute the strain energy release rates for different configurations. The appearance of 'partial' cracks during oxidation is interpreted by considering the stability of propagation across the different layers. Different cracking scenarios involving 'partial' and 'through-ply' cracks are modeled as well as their interaction during the evolution of the mechanical loading.

These results provide some insight on the mechanisms leading to crack creation under combined mechanical and environmental loading [3]. Further developments include the experimental determination of the strain energy release rate of the oxidized composite [4] and the strong coupling of the FFM approach with a diffusion/reaction model in order to account for the enhanced diffusion generated by crack creation for long aging times.

- [1] J. A. Nairn, S. Hu. *Damage Mechanics of Composite Materials*, chapter Matrix Microcracking, pages 187–243. Elsevier Science, 1994.
- [2] G. Lubineau, D. Violeau, P. Ladevèze. Illustrations of a microdamage model for laminates under oxidizing thermal cycling. *Composites Science and Technology*, 69, 3-9, 2009.
- [3] F. Daghia, F. Zhang, C. Cluzel, P. Ladevèze. Effect of oxidation on the transverse cracking behavior of  $[90/0]_s$  carbon-epoxy laminates. *Submitted*.
- [4] F. Daghia, C. Cluzel. Experimental study of the propagation of delamination under oxidizing environment and mechanical loading. In *ECCM16*, Seville, 2014.

# Micro-mechanical analysis of the *in situ* effect in polymer composite laminates

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The effect of ply thickness on the onset of ply damage (transverse cracking) and delamination is extremely important for the prediction of the mechanical response of laminated composites. This is of particular significance when dealing with the most recent spread-tow, ultra-thin grades, where dry ply thicknesses can be as low as 0.02 mm, considerably below the standard for carbon fibre-reinforced polymer low grade tapes.

When embedded in a multidirectional laminate, the actual strengths and the crack density in the transverse plies are not only higher than those measured in unidirectional coupons, due to the constraints imposed by the neighbouring plies, as they reportedly increase with decreasing ply thickness [1, 2, 3]. For very low ply thicknesses, such as those of the thinner spread tows, fracture mechanics predictions of the *in situ* strengths [1] tend to infinity; therefore, an understanding of the *in situ* effect for such ply thinness is of crucial importance, namely for the development of physically meaningful analytical models applicable at this scale. However, experimental analysis based on *in situ* observations of very thin plies are extremely complicated, not only because it becomes relatively hard to identify any transverse damage at this scale, but also because the applied strains needed to develop such damage mechanisms are so high that conduct to failure (or severe damage) of the constraining material. A possible way through for the understanding of the *in situ* effect is the use of computational micro-mechanics.

In the present work [4], a micro-mechanical finite element model of a composite sublaminate is proposed to study the mechanical response of ultra-thin plies, consisting of a representative volume element of a 90° thin lamina in-between two homogenised  $\pm \theta^{\circ}$  plies. Random fibre distributions, materially and statistically equivalent to real distributions [5], are analysed. A three-dimensional computational micro-mechanics framework [6, 7], with a special focus on the elastic-plastic and damage constitutive behaviours of the matrix and on the response of the fibre-matrix interface, is used in the present analysis. Varying the 90° ply thickness, it is possible to assess its effect on the mechanical response of laminated composites — the *in situ* effect. The proposed framework is able to accurately represent the micro-mechanical response of ultra-low grades, including (i) the mechanics of transverse cracking onset and propagation, (ii) the constraining effect observed in the laminae embedded in multidirectional laminates, (iii) the gradual, slow stress relaxation and progressive transverse cracking observed in very thin plies and consequent increase of the crack density, (iv) the reduction in crack-opening displacement of the transverse cracks with ply thinness, (v) the formation of thin necks of matrix material around the regions where interfacial damage is more pronounced, and (vi) the *in situ* strengths.

- [1] P.P. Camanho, C.G. Dávila, S.T. Pinho, L. Iannucci, P. Robinson. Prediction of in situ strengths and matrix cracking in composites under transverse tension and in-plane shear. *Composites Part A: Applied Science and Manufacturing* 37, 165-176, 2006.
- [2] H. Saito, H. Takeuchi, I. Kimpara. Experimental evaluation of the damage growth restraining in 90° layer of thin-ply CFRP cross-ply laminates. *Advanced Composite Materials* 21, 57-66, 2012.
- [3] H. Saito, H. Takeuchi, I. Kimpara. A study of crack suppression mechanism of thinply carbon-fiber-reinforced polymer laminate with mesoscopic numerical simulation. *Journal of Composite Materials* 48(17), 2085-2096, 2014.
- [4] A. Arteiro, G. Catalanotti, A. R. Melro, P. Linde, P. P. Camanho. Micro-mechanical analysis of the *in situ* effect in polymer composite laminates. *Composite Structures* 116, 827-840, 2014.
- [5] A.R. Melro, P.P. Camanho, S.T. Pinho. Generation of random distribution of fibres in long-fibre reinforced composites. *Composites Science and Technology* 68, 2092-2102, 2008.
- [6] A.R. Melro, P.P. Camanho, F.M.A. Pires, S.T. Pinho. Micromechanical analysis of polymer composites reinforced by unidirectional fibres: Part I - Constitutive modelling. *International Journal of Solids and Structures* 50, 1897-1905, 2013.
- [7] A.R. Melro, P.P. Camanho, F.M.A. Pires, S.T. Pinho. Micromechanical analysis of polymer composites reinforced by unidirectional fibres: Part II - Micromechanical analyses. *International Journal of Solids and Structures* 50, 1906-1915, 2013.

# A mesh independent simplified cohesive segment method to model matrix cracking in composites

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In this work, a technique is presented to simulate transverse matrix cracking in composites using a novel simplified cohesive segment method[1]. This method is implemented in Abaqus/Explicit as a 3D user defined element (VUEL). It relies on a cohesive traction-separation formulation to represent cracks which are inserted within elements in a completely solution dependent manner and the subdomains on either side of the crack segment are updated using a simple remeshing rule operating at an element level. Because matrix cracks in composites always occur parallel to the fibres, there is no need to determine the direction of crack growth during solution, which saves computational cost. This formulation is compared with authors' earlier work of modelling composite failure using continuum damage models[2], and shows that it is able to circumvent the well-known problem of mesh orientation bias of damage growth, which is otherwise present in continuum damage modelling. A significant mechanism of failure in composite materials is by delaminations joining up through adjacent matrix cracks. Therefore test cases are presented where this matrix crack model is combined with traditional cohesive elements to model delamination failure between plies, and shows that they can achieve a much better interaction between matrix cracks and delaminations compared to the continuum damage modelling approach.



Figure 1. Sequence of matrix crack growth in a regular mesh (post processed to display user elements)

## Acknowledgements

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- [1] L. F. Kawashita, A. Bedos, and S. R. Hallett, "Modelling mesh independent transverse cracks in laminated composites with a simplified cohesive segment method," *CMC-Computers, Materials & Continua*, vol. 32, pp. 133-158, 2012.
- [2] S. Mukhopadhyay, M. I. Jones, and S. R. Hallett, "Modelling of out-of-plane fibre waviness; tension and compression tests," in IV ECCOMAS thematic conference on mechanical response of composites, Ponta Delgada, 2013.

# A computational approach to the ultimate failure of unidirectional ceramic-matrix composites

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Ceramic-matrix composites (CMC) exhibit a non-linear mechanical behavior and a quasi-brittle failure despite being made of brittle ceramic matrix and fibers. This peculiar behavior originates in the creation of multiple matrix cracking and debonding at the fiber/matrix interface. The matrix cracking process reaches a saturation density before the ultimate load. The ultimate failure of the composite results from the accumulation and interaction of successive fibre breaks. The fiber break failure process is driven by the interplay between the random distribution of the fibers strength and the load transfer between the broken fibers and the remaining ones. A large number of analytical models have been dedicated to the estimation of the tensile strength of unidirectional CMC. These models generally consider a regular periodic fiber arrangement and suppose either a global load sharing (GLS), i.e. the load previously carried by a broken fiber is equally redistributed over all the remaining intact fibers, or a local load sharing (LLS) where the redistribution is restricted only to its first neighbors. However, it was demonstrated [1] that the choice of a particular load sharing mechanism could significantly influence the estimated strength and its associated scatter. It was also proven that an imperfect local load sharing (ILLS) mechanism, in-between GLS and LLS, was able to better capture the experimental strength distribution.

The aim of this work is to overcome the *a priori* choice of any particular fiber arrangement and load sharing mechanism by performing direct numerical simulations of the fiber breaking process in unidirectional CMC. We propose a computational approach based on realistic finite element models of unidirectional CMC constructed from optical micrographs of real CMC materials (cf. Fig. 1). As we focus our attention to the fiber break process, the damage state reached at saturation is initially introduced into the meshes. A strain-controlled tensile test is then simulated by gradually applying an axial displacement to the specimen. At each step, the mean axial stress of the fibers is computed and compared to their strength which is classically modelled using a Weibull distribution. The mechanical equilibrium is repeatedly checked after each fiber break as it could lead to further breaks due to load transfer. The outcomes of these computations include the strain-stress curve and stiffness evolution from matrix crack saturation to the ultimate failure, but also the precise scenario of fibre breaks, and the evolution of the matrix cracks opening, fiber break cluster size as well as the critical level of fibre break leading to a



FIGURE 1 – Example of a FE model of an unidirectional CMC used for the direct numerical simulation of the fiber breaking process.

catastrophic failure. The effect of the debonding length, fiber arrangement and strength is discussed, and the resulting strength distributions are compared to the ones obtained with GLS and LLS.

We expect these results will lead to a better understanding of the ultimate failure of CMC and will provide evidences to support the choice of a particular load sharing mechanism.

## Références

[1] V. Calard, J. Lamon. Failure of fiber bundles. *Composites Science and Technology*, 64, 701–710, 2004.

# A Phenomenological Damage Model for Elastoplastic Behavior of Particle Reinforced TRIP-Steel Matrix Composites based on Micromechanical Simulations

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This paper proposes a phenomenological modeling approach for particle reinforced TRIP-steel matrix composites. The development of the model is based on micromechanical simulations of the composite material, which are used twofold: Firstly, they help to formulate requirements on phenomenological modeling on the mesoscale. Secondly, they enable a calibration of the mesoscale model to the results of the micromechanically simulated composite behavior.

Several micromechanical mechanisms influence the behavior of metal matrix composites during loading. On the one hand, a high hardening of the composite is obtained due to particle reinforcement effects [1]. On the other hand, deformability is limited by favoured ductile damage processes in the matrix as well as rupture of the particles and interfaces. To improve the ductility of the composite, steels showing the TRansformation Induced Plasticity (TRIP) effect have been identified as possible matrix materials with large forming capability as well as a high hardening [2].

In the present study, the impact of interface damage on the microscopic and mesoscopic behavior of the TRIP-steel matrix composite is investigated. A cell model is used to analyze the microstructure of the composite as a single elastic particle embedded in a TRIP-steel matrix. A low volume fraction of reinforcements (5 %) is considered. To describe the inelastic matrix behavior of the TRIP-steel, Hallberg's [3] constitutive approach is utilized that incorporates the austenite to martensite phase transformation, which is responsible for the TRIP-effect. The interface and its failure are modeled by cohesive zone elements using a bilinear traction-separation law.

The cell model is numerically implemented by means of the finite element method. For uniaxial tension and compression loading, mesoscopic quantities of interest are evaluated by means of averaging procedures: yield curves, martensite evolution and an integral damage variable to define interface failure.

The proposed modeling approach on the mesoscale can be summarized as follows: The undamaged composite is seen as a homogenized bulk material, where the particle reinforcement effect is cast into the overall hardening behavior, following [4]. Assuming that detached particles act as nucleation sites for voids, a ductile damage mechanism caused by void growth and coalescence can be concluded. Also the influence of damage on the mesoscopic martensite evolution should be captured correctly. Therefore, Hallberg's [3] TRIP-steel model is expanded by damage terms known from the porous metal plasticity model of Rousselier [5]. Thus, the evolution of porosity influences plastic yielding of the composite. The additional damage progress of interface failure is formulated by a mesoscopic stress based criterion. An associated damage variable develops, obeying a Weibull distribution. This damage variable is furthermore used to formulate a void nucleation term entering the evolution equation of porosity. In order to match large differences of yielding with respect to the loading direction, the plastic yield function is considered to be dependent on the third stress invariant. This way, a high tension-compression asymmetry arises through the damage processes, the direction dependent phase transformation and the yielding of the TRIP-steel matrix itself.

The phenomenological model is calibrated to the cell model results of the TRIP-steel matrix composite. As a result, the phenomenological model can well reproduce the meso-scopic yielding, martensite evolution in the metal matrix and the progress of interface damage for monotonic tension and compression loadings.

- [1] N. Chawla and Y. L. Shen. Mechanical Behavior of Particle Reinforced Metal Matrix Composites. *Advances Engineering Materials*, 3(6):357-370, 2001.
- [2] H. Biermann, U. Martin, C. G. Aneziris, A. Kolbe, A. Müller, W. Schärfl, and M. Hermann. Microstructure and Compression Strength of Novel TRIP-Steel/Mg-PSZ Composites . Advances Engineering Materials, 11:1000-1006, 2009.
- [3] H. Hallberg, P. Håkanson, and M. Ristinmaa. A constitutive model for the formation of martensite in austenitic steels under large strain plasticity. *International Journal* of Plasticity, 23:1213-1239, 2007.
- [4] H. Zhang, K.T. Ramesh, and E.S.C. Chin. A multi-axial constitutive model for metal matrix composites. *Journal of Mechanics and Physics of Solids*, 56:2972-2983, 2008.
- [5] G. Rousselier. Dissipation in porous metal plasticity and ductile fracture. *Journal of the Mechanics and Physics of Solids*, 49:1727-1746, 2001.
# Translaminar fracture of WRM composite using Random Spring Network

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Fiber reinforced composites (FRC) are increasingly being used for structural applications in aerospace industry. To exploit their superior mechanical performance and associated cost benefits, it is of significance to develop a comprehensive understanding of the dominant failure modes: inter-laminar/delamination caused primarily due to matrix cracking and intra-laminar or translaminar fracture which has both matrix cracking and significant fiber breakage [1]. In a recent study on translaminar mixed-mode fracture of plain weave glass/epoxy composite, Boyina et al. [2] showed that under conditions of higher mode-mixity the macroscopic response prior to final failure is highly non-linear in spite of the inherent brittleness of the constituents. From fractography it was established that the fracture process has significantly higher fiber-matrix debonding along the transverse fibers which resulted in gradual energy dissipation prior to final failure due to fiber breakage along the crack path. Complex brittle materials such as composite materials have statistically distributed mechanical as well as fracture properties and the fracture process in them involves micro-cracking at multiple locations, interactions between these cracks and their growth leading to final failure which is hard to model using conventional fracture mechanics theory. In the present study a statistical approach of [3] is taken by discretizing the fracture specimen domain into a two-dimensional square spring network lattice as shown in Figs. 1(a) and (b) respectively. Each lattice point is connected to the nearest and next-nearest neighbors through linear spring forces that resist both change in bond length as well as change in bond angles as shown in Fig. 1(c). Any spring when



Figure 1: Modelling WRM composite using random spring network.



Figure 2: Macroscopic response from simulations and experiments of modified compact test specimen for a glass-epoxy plain weave composite material.

stretched beyond a threshold failure strain is considered broken and the elastic force due to that spring becomes non-existent. In the fracture simulations, displacements are applied at the top boundary of the lattice while the bottom boundary is held fixed. For every incremental displacement, position of each lattice point is updated iteratively based on the integration of its equation of motion using a velocity-verlet algorithm with an additional damping force included to expedite convergence to an equilibrium configuration. Condition of failure of bonds is applied and the network is allowed to converge again before the next increment in displacement is applied till final separation of the network as shown in Fig. 1(d). The complex heterogeneity of the WRM composite is modeled by defining two types of bonds: stiff or soft. Combinations of these bonds are used to identify four different regions: region that has high stiffness in both transverse and longitudinal direction, region that is stiffer in longitudinal direction but softer in transverse direction, region that is stiffer in transverse but softer in longitudinal direction and region which has lesser stiffness in both the directions. The relative proportion of these regions is assumed for any analysis and the lattice sites belonging to each region is then randomly assigned a location in the domain. Macroscopic response from simulations in Fig. 2 are for different combinations of threshold failure strain for stiff bonds ( $\varepsilon_s$ ) and the failure strain ratios  $(\varepsilon_w/\varepsilon_s)$ . It can be seen that while the peak load of the macroscopic response can be well approximated by a homogeneous network, the post-peak behavior for the combination with failure strain ratio at 1.0125 has significantly more dissipation during the growth of the initial crack and therefore compares well with the experimental data.

- [1] M.J. Laffan, S.T. Pinho, P. Robinson. Mixed-mode translaminar fracture of CFRP: failure analysis and fractography, *Compos Struct*, 95, 135-141, 2013.
- [2] Dhatreyi Boyina, Anuradha Banerjee, R. Velmurugan. Mixed-mode translaminar fracture of plain-weave composites, *Composites Part B: Engineering*, Volume 60, 21-28, April 2014.
- [3] C. Urabe, S. Takesue. Fracture toughness and maximum stress in a disordered lattice system, *Phys. Rev. E*, 82, 016106, 2010.

# Modelling multiple delamination and intralaminar cracks using a single-layer shell approach

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The introduction of fibre reinforced polymers in the automotive industry is strongly dependent on accurate and efficient CAE tools to predict the correct energy absorption in car crash analyses. From experimental observations during axial crushing, it is obvious that in order to obtain good predictability in the simulations, delamination needs to be accounted for. However, due to industrial restrictions on the total simulation time of full scale crash analyses, detailed modelling of each ply as represented by separate elements through the thickness – to promote delamination modelling by cohesive interface elements – is impossible.

In order to provide an alternative approach, we have recently proposed an enriched XFEM shell element formulation with the benefit that multiple propagating delamination cracks can be kinematically represented independently of the finite element mesh. Hence, a structural model of a thinwalled laminate can thereby initially be built up by a single layer of shell elements through the thickness. During loading, the model is then enriched locally (and adaptively) in critical areas where delamination is predicted.

However, as is well known, delamination is far from the only relevant failure mechanism which needs to be considered in the simulation of crash-like events. In combination with multiple delamination cracks, severe intralaminar failure (transverse matrix failure, fibre kinking, matrix shear failure etc.) is observed in the highly strained areas of the structure. These mechanisms are often described in the literature via progressive damage models based on an embedded cohesive zone approach in which the cohesive degradation behaviour is 'smeared' over the element experiencing damage evolution in a way such that pathological mesh dependence (which is always present for a local continuum damage model) should be avoided. In contrast to this, some authors have with success modelled intralaminar crack propagation explicitly by using the XFEM (or phantom node method), although for the case where each ply is discretised explicitly by separate shell or solid elements.

In this contribution, we investigate the potential of further extending the previously developed enriched shell element capable of representing multiple delaminations to also handle intralaminar crack propagation without having to explicitly model each ply by separate (shell) elements through the thickness. Instead, the addditive enrichment for multiple delaminations is combined with additional enrichment functions confined only to individual plies, making it possible to represent individual intralaminar cracks with different orientations in different plies.

To give a short illustration of the capabilities of the model, we consider a purely theoretical case in which a  $[0]_7$  laminated beam, modelled by a single layer of shell elements through the thickness (in total 6 x 40 quadratic triangular shell elements) and with a crack pattern as indicated in Figure 1a. Thus, there is a combination of short delamination cracks with perpendicular intralaminar cracks which together conform to a crack propagating along the entire beam, but which at certain points goes through the plies in a staircase manner. Furthermore, the beam is loaded by a traction load *t* along its upper rightmost edge. The current example has been chosen for its simple analytical solution (assuming Euler Bernoulli beam theory) if transverse strains are disregarded. The intention is to show that the model is able to describe (kinematically) complex crack patterns involving both delamination and intralaminar cracks with good accuracy.

The deformation pattern obtained in the shell analysis is shown in Figure 1b. By comparing the results from the shell analysis with the analytical solution (via Euler-Bernoulli beam theory) the difference in maximum end displacement is less than 0.2 % which verifies that the kinematics in the case of combined delaminations and intralaminar cracks is correct.



Figure 1: Sketch and deformed shape of laminated beam with a staircase crack (a combination of delamination and intralaminar cracks). Please note that only one layer of shell elements is used in the analysis

The results indicate that the level of detail in the present approach is such that individual delamination and/or intralaminar cracks and can be analysed accurately where present. Moreover, the level of detail (and computational cost) of the model is only increased in areas where crack initiation and propagation actually occurs thereby reducing the computational effort required in large scale analyses. Thus, we see a potential for this modelling concept in analyses in which computationally efficiency is of major importance, such as car crash analyses.

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### A New Mixed DEM/FEM Approach to Model Advanced Damage of Reinforced Concrete

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The discrete element method (DEM) is a powerful alternative to the finite element method (FEM) when advanced damage states and failure of concrete have to be studied (Hentz et al. 2004). Indeed, DEM allows very easily obtaining realistic macro-crack patterns and material fragments due to its discontinuous nature.

To model reinforced concrete structures, it is also necessary to take into account the steel reinforcement and its interaction with concrete. It is possible to model the rebars within DEM framework by using aligned discrete elements and special beam-like links as proposed in (Rousseau 2009). However, to be able to consider complex grid patterns of rebars used in real engineering structures, the use of DEM-type model for steel is not optimal for many reasons: difficult mesh construction, poor numerical performance, etc. In this contribution, a new steel-concrete bond model is proposed to link in an appropriate way DEM concrete to FEM rebars. This model uncouples the normal and the tangential responses of the steel-concrete interface and allows choosing a suitable form of constitutive laws for each component. Thus, they can be fitted separately in accordance with experimental data. The normal and tangential bond laws are described.

The proposed approach is validated by simulating the well known Brazilian indirect tension test, and the Nooru-Mohamed shear-tension test with a good reproduction of curved cracks. Then, the whole approach is applied to simulate the steel-concrete tie in tension. Realistic cracking process and a correct load transmission between the rebar model and the concrete model are obtained. Impact simulations are also presented.

- Hentz, S., Daudeville, L. and Donzé, F.V. Discrete Element Modelling of Concrete Submitted To Dynamic Loading at High Strain Rate. *Comput. Struct.*, 82, 2509-2524, 2004.
- J. Rousseau, Modélisation numérique du comportement dynamique de structures sous impact sévère avec un couplage éléments discrets / éléments finis. Thèse de doctorat, Université Joseph Fourier, Grenoble, 2009.

# Dynamic effects in unidirectional fiber-reinforced composites: a peridynamic analysis

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Modeling of damage and failure in fiber-reinforced composites (FRCs) is a very challenging problem because of the complexity of stress and strain states induced by microheterogeneities and elastic and fracture toughness anisotropy. Some recent peridynamic (PD) models for FRCs (see, e.g. [1], [2]), which start from a homogeneous representation of these materials, have been attempting to simulate dynamic crack growth and failure in FRCs. It has been an open problem whether homogenized models can lead to crack localization, which is experimentally observed in failure of FRCs. In this talk I will review some existing peridynamics models and discuss an improved version of one of them. I will then compare the results these models give for several tests cases: impact and intersonic crack propagation in unidirectional FRCs (see [3]), and dynamic crack growth from quasi-static or dynamic loading of unidirectional FRCs.

We find that PD models that try to mimic the macroscopic slowness curve (tension surface) at the PD microscale do not lead to failure modes seen in some experimental tests on FRCs. On the other hand, PD models that assume only two types of micro-bonds, similar to the actual two-phase composition of the FRC, are capable of reproducing, accurately, failure modes and crack propagation speeds seen in experiments. In particular, we discover an interesting phenomenon: a strong strain-rate dependence for the modes of failure in a unidirectional 45° FRC. In the quasi-static loading regime, we observe that the crack initiates and grows perpendicular to the loading direction, through the matrix, over the fibers, and without breaking them. Under high strain-rate loading, the crack initiates along the 45° fiber direction, leading to splitting-mode (fiber-matrix delamination) failure of the composite. We hope that experimentalists will attempt to verify our numerical predictions.

#### References

 W. Hu, Y. D. Ha, F. Bobaru. Modeling Dynamic fracture and Damage in a Fiber-reinforced Composite Lamina with Peridynamics. *International Journal for Multiscale Computational Engineering*, 9(6), 707-726, 2011.

- [2] M. Ghajari, L. Iannucci, P. Curtis. A Peridynamic Material Model for the Analysis of Dynamic Crack Propagation in Orthotropic Media. *Computer Methods in Applied Mechanics and Engineering*, 276, 431-452, 2014.
- [3] D. Coker, A.J. Rosakis. Experimental Observations of Intersonic Crack Growth in Asymmetrically Loaded Unidirectional Composite Plates. *Philosophical Magazine A*, 81(3), 571-595, 2001.

- chapter 8 -

# **66** MS9: Industrial applications

**99** 

# Influence of residual stresses on lifespan and crack path assessment of a disk blade connection

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During the last 15 years, significant progresses in crack propagation simulation of brittle materials have been observed thanks to the introduction of innovative numerical methods. Among them, there is the eXtended Finite Element Method (XFEM). XFEM enables the cracks to cut the finite elements so that the mesh must not be conformed to the crack. Consequently, the remeshing strategy is facilitated. Indeed, cracks are implicitly represented thanks to two level-sets functions. In linear elasticity, crack propagation is based on the computation of the Stress Intensity Factors (SIFs) and the use of well-chosen crack propagation laws depending on the context. Level-sets are then updated to model the crack propagation.

This study focuses on the lifespan assessment of a technological sample representative of a disk blade connection. Crack propagation rates and crack paths are experimentally obtained from a series of tests under various stress and temperature conditions. A first assessment of the crack path and lifespan with XFEM is obtained without considering any residual stresses. Good agreement can be noticed on the lifespan. However, small differences in the crack shape can be observed.

These differences possibly come from the presence of residual stresses near the free surfaces induced by shot peening on the sample. Their influence on the crack path and lifespan is investigated. Various residual stresses distributions are tested based on experimental data. In each case, numerical results are compared with experimental observations. These comparisons highlight that the residual stresses can explain the decrease of the crack propagation rate at the vicinity of the external layers of the material.

# Damage evolution in pearlitic steel specimens under tension by means of X-ray computed tomography

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Steel fracture mechanisms have been studied extensively over the past decades, but some aspects still remain unclear. Pearlitic steel, commonly used for manufacturing prestressing steel wires, presents an unusual fracture surface when tested under tension; a cylindrical specimen shows a flat surface with a dark region centered in the specimen surrounded by a bright area. When these zones are observed by means of a scanning electronic microscope, the mechanism of nucleation and growth of microvoids can be identified in the dark region.

On the other hand, one of the most widespread models typically used to reproduce the behaviour in metals is the Gurson-Tvergaard-Needleman (GTN) model [1, 2], which is based on the classic theory of nucleation, growth and coalescence of microvoids. These models have proved to be successful through the years and are able to reproduce the macroscopic damage behaviour of many metals.

In this contribution, the authors analyse the evolution of the internal damage by using X-ray computed tomography on a 3mm-diametre specimen. The specimen is tested in consecutive loading steps, after each of which it is unloaded and analysed with a X-ray tomograph. This procedure helps to identify the evolution of damage developed inside the specimen at predefined strain levels. Finally, these results are compared with the numerical results provided by the GTN model.

- [1] Gurson, A.L. Continuum theory of ductile rupture by void nucleation and growth: Part I – Yield criteria and flow rules for porous ductile media. *J. Eng. Mater. Tech.*, January, 1977.
- [2] Tvergaard, V., Needleman, A. Analysis of the cup-cone fracture in a round tensile bar. *Acta metall.*, vol. 32, number 1, 157–169, 1984.

# Finite Element Analysis of Ductile Fracture during Shearing Process of Shield Slot Plate for Molten Carbonate Fuel Cell

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In this study, the shearing process of shield slot for molten carbonate fuel cell was numerically investigated by finite element analysis. The shield slot plate for molten carbonate fuel cell is composed of open trapezoidal shape slots, manufactured by shearing process, to support the electrode and form the gas flow channel.

The three dimensional finite element analyses were conducted by using commercial software Abaqus/Explicit with the user material subroutine VUMAT with various ductile fracture criteria such as Forming Limit Diagram, normalized Cockroft-Latham, and Modified Mohr-Coulomb criteria [1].

For more accurate results, the flow stress obtained from tensile test was corrected by comparing load-stroke curves between experiment and finite element simulation of tensile tests after onset of necking [2]. The critical damage value was also determined using the corrected flow stress curves.

Finally, the simulation results of shearing process was compared with experimental results to compare the feasibility of ductile fracture criteria on shearing process of shield slot for molten carbonate fuel cell.

- [1] Yuanli Bai, Tomasz Wierzbicki, Application of extended Mohr-Coulomb criterion to ductile fracture, International Journal of Fracture, 161, 1-20, 2010
- [2] Seong-Hoon Kang, Young-Seon Lee, Ho Won Lee, Evaluation of flow stress and damage index at large plastic strain by simulating tensile test of Al6061 plates with various grain sizes, International Journal of Mechanical Sciences, 80, 54-61,2014.

# Current status in fuel performance codes of cracks and damage modelling

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Fuel elements used in Pressurized Water Reactor reactors are made of a stack of uranium dioxide pellets enclosed in a zirconium alloy cladding.

Modelling the fuel rod behaviour during irradiation in normal and off-normal operating conditions, requires to take into account numerous coupled phenomenon. In particular, crack propagation and damage mechanics, in the fuel pellet and in the cladding, have been increasingly used in CEA simulations [1, 2, 3]. One important aspect of those simulations is to cope with the wide range of temperatures, strain rate, and stress levels encountered.

Modelling of crack in fuel rod simulation can be illustrated in various ways:

- At low temperature, oxide fuel pellets are indeed brittle especially at low temperature. Because of the temperature profile in a fuel rod, cracks are initiated at the outer edge of the pellet and tend to propagate inwards as the power increases. Numerical simulations of this early stage of the fuel irradiation, as illustrated on Figure 1, leads to a crack pattern qualitatively close to the experimental one.
- During RIA experiments in the Cabri reactor, ductile failure of the cladding was observed starting from cracks coming from zirconium hydride blisters [4]. Figure 2 illustrate some simulations results obtained for this particular problem.

The aim of this paper is to expose the current status of the crack modelling used in the CEA Alcyone fuel performance code. Numerical aspects, including the treating of snap-back due to the quasi-static assumptions, of this modelling will be discuss in depth.

#### References

B. Michel, J. Sercombe, and G. Thouvenin. A new phenomenological criterion for pellet-cladding interaction rupture. *Nuclear Engineering and Design*, 238(7):1612–1628, July 2008.



Figure 1: Initial fuel pellet fragmentation



Figure 2: Modelling the cladding failure during an RIA transient

- [2] Bruno Michel, Jérôme Sercombe, Gilles Thouvenin, and Rémy Chatelet. 3d fuel cracking modelling in pellet cladding mechanical interaction. *Engineering Fracture Mechanics*, 75(11):3581–3598, July 2008.
- [3] Bruno Michel, Christelle Nonon, Jérôme Sercombe, Frédéric Michel, and Vincent Marelle. Simulation of pellet-cladding interaction with the PLEIADES fuel performance software environment. *Nuclear technology*, 182, June 2013.
- [4] Matthieu Le Saux. Comportement et rupture de gaines en Zircaloy-4 Detendu vierges, hydrurées ou irradiées en situation accidentelle de type RIA. PhD thesis, École des Mines de Paris, October 2008.

# Investigation in rolling contact fatigue crack growth in rails of SNCF network

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Due to the repeated passages of the wheels, rolling contact fatigue cracks can appear in the surface or subsurface of the rails. These defects, such as squats and head-checks, can propagate and lead to the rail fracture. If a fracture of the rails occurs, it could cause not only a heavy economic impact (delayed trains, maintenance costs), but also passenger safety (derailment). In order to avoid the rail fractures, French National Railroad Company (SNCF) performs a strict method with both corrective (consolidation or removing of damaged rails, local deep grinding...) and preventive (initial and periodic grinding, periodic monitoring by ultrasound...) maintenance strategy.

This paper attempts to investigate the rail maintenance against rolling contact fatigue by combining the numerical simulations (from train-track dynamics to fatigue crack growth simulations) with the analysis of defects observed in our network (SNCF's database analysis). Firstly, a parametric numerical study is performed to better understand the effect on the fatigue phenomenon of various factors such as the wheel-rail contact forces, the residual stresses, the rail bending and track stiffness, the localization of the cracks (over or between sleepers), the crack path.... A quick description is given of the models and simulation tools that were developed within the consortium IDR2 gathering SNCF, RATP the Parisian metro operator and TataSteel, one of the major rails manufacturers. Secondly, the observed rail defects (squats and head-checks) on a high speed line, collected from different SNCF's databases, are analysed, taking account of the track geometry, the rolling stock, the traffic and the historical maintenance operations (grinding, stuffing, removing of damaged rail). Finally, some analysis focusing on rail defects (density and evolution) around turnouts and bridges will be discussed.

- [1] B. Trollé. Simulation multi-échelles de la propagation des fissures de fatigue dans les rails. Thèse CIFRE SNCF LaMCoS, 2014.
- [2] B. Trollé et al. 2D fatigue crack propagation in rails taking into account actual plastic stresses. Engineering Fracture Mechanics 123 (2014) 163-181.
- [3] M.L. Nguyen-Tajan and C. Funfschilling. A numerical modelling strategy for the rolling contact fatigue analysis of rails. WCRR 2011.

# Modeling Failure Progression in Structures Informed by Demolition and Forensic Experience

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In demolition engineering and forensic investigations we have the opportunity to observe how structures fail. We use the information we have observed while demolishing structures and after collapses to inform how we model structural failures. From demolition engineering we have learned about the uncertainties associated with material failure. When similar structures are damaged in similar ways under similar loads, they can fail in different mechanisms. We have gained insights into when and how a failure propagates and when it remains local.

In forensic engineering practice analytical models are constructed to recreate the collapse and to confirm that the cause of the collapse has been properly identified.

In this paper several case studies of concrete and steel structures show how different sets of assumptions and modeling approaches are used depending on the type of failures to be modeled and how inappropriate assumptions can lead to erroneous conclusions. For example: understanding post-buckled capacity is essential to capturing the I-35W steel bridge collapse, and fiber beam models used in seismic application can capture cyclic behavior of concrete members without the need for simulating the precise crack propagation.

### Development of xfem tools in industrial environment

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The development of the x-fem methodology within the industrial environment of Salomé Méca free software started ten years ago with a collaboration in between EDF and Professor Nicolas Moës of the Ecole Centrale de Nantes. At the beginning of the project the industrial need was to be able to perform parametric studies of defaults with different shapes and positions in the rotor of a shaft line. This initial need moved towards the one of being able to simulate crack propagation.

The advantages of X-FEM are:

- an ease to define the default geometry and then to introduce it in a flawless model, with the help of the level set formalism [1], which is really appreciated by the engineers,
- if done properly, the treatment of system conditioning when the default approaches the nodes of a mesh or is very near from a free surface [2,3], not possible to our knowledge with other methods such as re-meshing in the latest situation,
- if done properly, an optimal rate of convergence of 1 with linear elements and two with quadratic elements, which is not the case for FEM with a crack meshed [4],
- if done properly, incompatibility interface unknowns treatment to avoid spurious oscillations of Lagrange multipliers when crack closure is considered [5], for instance with a cohesive law [6,7].

Its main drawbacks are:

- the creation of new elements for which you have to derive the stiffness matrix which leads to double the implementation development efforts in your software, with some extra costs for maintenance: for each specific physics its x-fem counterpart has to be introduced which do not occur with re-meshing techniques,
- additional work to apply boundary conditions or impose displacement controlled solutions, in case of instability for example.

On-going works at EDF concern now the development of quadratic xfem elements to reduce the error level, with important efforts focalized on solving conditioning issues [3]. As you go for quadratic, you will also need to reduce the number of integration points which is skyrocketing due to quadratic subdivisions. For that purpose we used a preceding work [8] conducted on integration in order to take into account plasticity with x-fem elements, and which allows reducing drastically the number of integration points to the one of standard fem elements.

Another research axis is concentrated on propagation algorithms. Our initial algorithms [4]

were improved and are now based on fast marching methods so as to be robust with increased performance.

Finally, also to increase performance, discrete potential methods were applied to a cracked model with x-fem. A regular problem without a crack can be solved and reduced complementary systems involving the degrees of freedom around the crack are solved to recover the solution of the problem with crack. Applying finite difference or fft to the regular problem can help us reduce the computational time [11].

- [1] A. Gravouil, N. Moës, and T. Belytschko, Non-planar 3D crack growth by the extended finite element and level sets Part II: Level set update. International Journal for Numerical Methods in Engineering, 53:2569–2586, 2002.
- [2] N. Moës, E. Béchet, and M. Tourbier, Imposing Dirichlet boundary conditions in the extended finite element method. Int. J. Num. Meth. Engng, 67:1641-1669, 2006.
- [3] M. Ndeffo, P. Massin and N. Moës, Extended finite element method with 3D quadratic elements: integration and conditioning issues, 5<sup>th</sup> European Conference of Computational Mechanics, July 20-25, 2014, Barcelona, Spain.
- [4] P. Laborde, J. Pommier, Y. Renard, M. Salaün. High-order extended finite element method for cracked domains, International Journal for Numerical Methods in Engineering, 64(3):354–381, 2005.
- [5] G. Ferté, P. Massin, N. Moës, Interface problems with linear or quadratic X-FEM: design of a stable multiplier space and error analysis, Int. J. Num. Meth. Engng,, 100(11): 834–870, 2014.
- [6] E. Lorentz. A mixed interface finite element for cohesive zone models. Computer methods in applied mechanics and engineering, 198:317-320, 2008.
- [7] P. Massin, G. Ferté, N. Moës, 3D crack propagation with x-fem cohesive elements, cfrac 2015.
- [8] A. Martin, J.B. Esnault, P. Massin, About the use of standard integration schemes for X-FEM in solid mechanics plasticity, CMAME, 283:551-572, 2015.
- [9] D. Colombo, P. Massin. Fast and robust level set update for 3D non-planar X-FEM crack propagation modelling, Computer Methods in Applied Mechanics and Engineering, 200:2160-2180, 2011.
- [10] D. Colombo. An implicit geometrical approach to level-set update for 3d nonplanar x-fem crack propagation. Comp. Meth. Appl. Mech. Engng., 240:39-50, 2012
- [11] W. H. Woodward, S. Utyuzhnikov, P. Massin, On the Application of the Method of Difference Potentials to Linear Elastic Fracture Mechanics, submitted to IJNME.

# Modelling and Simulation of TB32 Crash Test of N2-W4-A Category Safety Barrier on Road Bend

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The paper examines the SP-05/2 outer barrier of N2-W4-A category (producer: Stalprodukt JSC, Bochnia) with the B-type guide bar, located on a horizontal concave arch to an accelerated traffic main road, with an allowable radius of road axis belonging to the range of 140–220 m. In the case of N2 restraint level, acceptance of TB11 and TB32 crash tests is required. The collision intensity level A reflects an impact of collision on vehicle occupants, measured by the acceleration severity index, ASI, and the theoretical head impact velocity, THIV, both calculated according to complex algorithms. The A level meets the conditions ASI  $\leq$  1.0 and THIV $\leq$  33 km/h. The working width W is a measure of barrier deformation. For level W4 distance W is  $\leq$ 1.3m. The barrier and the cap should preserve the integrity after collision with the vehicle. After repulse from the barrier, the vehicle must be included in an acceptable field of repulse.

In order to ensure accepting the modified TB32 crash test on a road bend, a rubber/foam/composite overlay, denoted with the code CFR2, has been designed, which was combined with the B guide with screw connectors using only the empty holes in the guide axis, with 2.00m spacing. The CFR2 covering system consists of glass–polyester composite segments, partly fulfilled with polyurethane foam. The segments are connected to the B-type guide bar with M16/80 bolts of class 8.8, using thick rectangular rubber pads produced of 70°ShA EPDM rubber and rectangular dip galvanized steel washers of type A (producer's specification). The covering is manufactured using hand lay-up technology. The CFR2 covering has a smooth front surface protected with gelcoat. The CFR2 overlay is flame retardant, resistant to weathering and required chemical effects, increases flexibility and strength of the barrier, reduces vehicle–barrier friction, and its estimated durability is 30 years.

The principle of action of CFR2 overlay is as follows. After hitting the barrier by the vehicle at an angle of 20°, elastic deformation of cap and rubber pads appears. There is time elongation of impact pulse. The overlay is closer to the guide B, and based on it. It is followed by progressive destruction of cap main components (composite shells and foam core), but to the extent that maintains the integrity of cap and bolted joints. The rubber pads soften the collision process. Friction between the vehicle and the gelcoat layer covering the cap is significantly reduced compared with the steel guide, which

facilitates easier sliding of vehicle on the barrier with the cap and contributes to securing the vehicle before skidding.

Test barrier sections of length of 60.00m constitute a straight barrier (SB) and a curved barrier (CB) of radius of 150m. This is the smallest radius of curvature of outer barrier, which allows a traffic speed of 110km/h. The barriers without (SB, CB) or with (SBC, CBC) the CFR2 covering system are under consideration.

The study develops a methodology for numerical modelling and simulation of unmodified (SB, SBC) and modified (CB, CBC) TB32 crash tests, including: advanced material models with failure of barrier and covering components, deformable bolt connections with limited load capacity, car tire pressure, contact with friction, gravitational load, possible erosion of finite elements, hourglass control, posts embedded in deformable solid subsoil, shell FE modelling of straight and curved barriers, reliable values of friction coefficients based on the authors' experimental tests, viscous damping of selected components of the system (barrier, covering, subsoil).

S235JR steel used to manufacture barrier parts has elastoplastic properties with isotropic hardening. A GFRP laminate reinforced with selected mat and fabric is modelled as an elastic-brittle material taking into account the Chang–Chang failure criterion. PUR S-42 polyurethane foam is reflected by the 'honeycomb' model available in LS-Dyna, used to model anisotropic foams. The subsoil, in which Sigma-100 posts are embedded, is reflected by a simple model used to foams and subsoil in a case where the material constants are not fully defined. The asphalt pavement and the shoulder are modelled as non-deformable surfaces. The Coulomb friction model is taken into consideration for all friction pairs.

Parts of SP-05/2 barrier section of length of 60m are modelled using four-node shell finite elements in the Belytshko-Tsay formulation. The subsoil is reflected by cylinders (one cylinder per post) with a radius of 1.00m and a height of 1.30m, using constant stress solid elements. The post/bracket/guide/pad screw connections and screw connections of guide segments with the CFR2 overlay are mapped by means of constrained generalized weld spot elements. Modelling deformable bolted joints of guiderail segments with each other and with the CFR2 overlay is based on 3D and 2D local finite element modelling. These connections are mapped by means of elastic-damping elements and used in the shell model of guide bar. Laminates in the CFR2 overlay are modelled using 4-node shell elements in the Belytshko-Tsay formulation.

The Dodge Neon car model, corrected respectively, was taken from the public library developed by the National Crash Analysis Center, USA. Crash tests were simulated using the finite element non-linear explicit code LS-Dyna v971. The results include all the collision parameters required by the EN 1317 standard.

The TB32 virtual crash tests have been conducted for the SP-05/2 barrier of N2-W4-A category, in four cases, i.e. for the barrier not equipped or equipped with CFR2 covering, located on a rectilinear road or on a road bend of radius of 150m. Effects of barrier curvature and covering equipment on the course of crash tests were investigated. It has been proved that the SP-05/2 barrier with the CFR2 overlay, located on road bends, provides acceptance of modified TB32 crash test.

# Numerical modelling of sheet-metal blanking using XFEM with ductile fracture

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Sheet-metal blanking is one of the most commonly utilized metal forming processes for mass production of steel-sheet laminations. The process has been well-characterized experimentally (see, for instance, references in [1]). However, current demands on geometrical accuracy of the laminations and high production speeds require transition of the process' optimization from the experimental to the numerical field. Typically, the finite element method is used to optimize the process.

As the sheet-metal blanking results in a total separation of the sheet metal, a numerical model requires not only a elasto-(visco)plastic material constitutive model, but also a fracture criterion and a crack propagation routine.

Several approaches to model the material failure in sheet-metal blanking has been proposed and analysed, but the material-constitutive-model-coupled approaches are generally considered more accurate as they allow gradual material degradation up to the initiation of an macroscopic crack.

As the finite-element-method-based crack propagation routines (e.g. remeshing, element deletion) often have problems with fast evolving discontinuities, the extended finite element method will be employed in this study.

The method has recently been extended to modelling plastic deformations and ductile crack initiation and propagation [2], [3], [4], [5], most of which are focused on a static analysis of ductile fracture initiation. A numerical analysis of sheet metal blanking at higher speeds, however, must also include the transient effects resulting from the process. Therefore, a time stepping scheme is required and the crack initiation and propagation algorithms, proposed by the authors, must be adjusted for time-related problems.

In this research, the extended finite element method with explicit time integration, coupled with the Lemaitre damage law and ductile crack initiation and propagation is presented. An alternative crack propagation algorithm is used, suitable for use in the context of time integration.

The approach is used to model the sheet-metal blanking process at different blanking speeds. Results are compared to the experimentally obtained data.

- [1] J. Slavič, Š. Bolka, V. Bratuš and M. Boltežar. A novel laboratory blanking apparatus for the experimental identification of blanking parameters. *Journal of Materials Processing Technology*, 214(2), 507 - 513, 2014.
- [2] Pourmodheji, R. and Mashayekhi, M. Improvement of the extended finite element method for ductile crack growth. *Materials Science and Engineering A*, 551, 255-271, 2012.
- [3] M. R. R. Seabra, P. Šuštarič, J. M. A. Cesar de Sa and T. Rodič. Damage driven crack initiation and propagation in ductile metals using XFEM. *Computational Mechanics*, 52, 161-179, 2013.
- [4] P. Broumand and A. R. Khoei. The extended finite element method for large deformation ductile fracture problems with a non-local damage-plasticity model. *Engineering Fracture Mechanics*, 112-113, 97-125, 2013.
- [5] J.P. Crété, P. Longère and J.M. Cadou. Numerical modelling of crack propagation in ductile materials combining the GTN model and X-FEM. *Computer Methods in Applied Mechanics and Engineering*, 275, 204-233, 2014.

# **Computational and Experimental Fracture Assessment of a Cracked Hand Wheel**

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Material imperfections and manufacturing faults coupled with severe service conditions can lead to flaw appearance in machine elements. Consequential crack growth and propagation can seriously affect integrity of such structures. This is particularly important when dealing with possible fracture of components that are used in transportation vehicles since catastrophic consequences can occur if crack growth susceptible components are used in vehicle assembly. Therefore, using fracture mechanics principles to assess design is almost necessary [1, 2, 3].

In this paper, computational and experimental fracture assessment of a cracked hand steering wheel was conducted. Multiple crack occurrence was noticed on a hand wheel mounted on a go-kart vehicle. Cracks emanating from a fastener hole were noted and, consequently, final fracture occurred. Fracture assessment of a cracked hand wheel was carried out using experimental and computational approach. Material thickness measurement and visual inspection was performed and material type was experimentally detected. Using computational fracture mechanics procedures based on the finite element analysis, numerical fracture assessment of the hand wheel was conducted. Variation of stress intensity factor with crack length is presented as a result of computational procedure. Possible causes of crack occurrence are outlined. Obtained results are valuable for predicting fracture behavior of the cracked hand wheel and can be taken as a reference for design and exploitation process of such component.

- [1] F.G. Gálvez Díaz-Rubio, D.C. Franco, V. Sánchez-Gálvez. Fracture strength of welded aluminium joints in commercial road vehicles. Engineering Failure Analysis, 13(2), 260-270, 2006.
- [2] A. Ktari, N. Haddar, H.F. Ayedi. Fatigue fracture expertise of train engine crankshafts. Engineering Failure Analysis, 18(3), 1085-1093, 2011.
- [3] M. Fonte, L. Reis, M. Freitas. Failure analysis of a gear wheel of a marine azimuth thruster. Engineering Failure Analysis, 18(7), 1884-1888, 2011.

# Contribution of advanced and non linear material behavior law for sizing industrial composite structures

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This article presents a review of the different models developed by Onera and transferred to our industrial partners as plugging in their FE code. It also explains the main philosophy adopted to propose these formulations by using thermodynamically consistent and continuum damage mechanics frameworks. This contribution focuses on the importance of taking into account the presence of damage on the prediction of the structural failure of a composite part. This general approach has been applied for different types of composite materials (ceramics and organic composite materials), for woven (2D or 3D) or UD composites and for different mechanical loadings (static, fatigue, low velocity impact).

The main difference between composite and metal materials is the effect of the presence of damage far from the final rupture. A further difference is the presence of several rupture modes, depending on the loading of the material. For example, for unidirectional composite laminates, the final rupture is the result of fibre breaks in case of in plane loading. For laminates with fibres oriented parallel to the loading direction (quasi-isotropic laminates for example), final rupture is caused by matrix cracks in case of in-plane loading ( $[\pm 45]_s$ ) and by delamination cracks for out-of-plane loading. This is the reason why the different Onera models take into account several damage mechanisms and several rupture modes. Hence, the main difficulties for the development of behavioural laws for composite material, are to propose generic models for the variety of composite materials, easy to be used by design engineers with a clear and easy model's parameter identification procedure, and applicable in an industrial context (software, computational cost...).

This article will focus on the application of such model to the sizing of composite structures, the advantage and the drawbacks of such approaches and the difficulties on the implementing such tools in commercial software. Onera has developed, during the last years, two generic models for the design of composite structures, OPFM for UD composite laminates [1], and ODM for woven composites[2]. All these models use the continuum damage mechanics framework in order to estimate the effect of the damage on the behaviour of the material. The thermodynamic framework of these approaches avoids a non-physical response of these models (evolution of the damage with no energy loss). The main idea of these models is to identify the different types of damage, their effect, and their evolution on a "mesoscale". Hence, for composite materials, the main damage in a

UD composite laminate consists of matrix damage, transverse cracking and local delamination; in a woven composite the main damage is the occurrence of microcracks. Type and orientation of the damage depend on the microstructure of the material. In Polymer Matrix Composites, the microcracks are oriented by the microstructure, while in ceramic composites microcracks are oriented by the loading. These damages have different effects and a different evolution for each type of composite material. The detailed estimation of the strain and stress inside the material allows the application of failure criteria based on these damage evolutions, and takes into account the eventual non linearity due to the viscosity of the matrix. Nevertheless, in cases with high stress gradients, satisfying the failure criterion doesn't imply the failure of the entire structure. It is then necessary to describe the progressive failure of the material by implementing a softening behaviour. This type of behaviour induces not also numerical difficulties but also complexities for establish a clear and simple identification procedure. Onera is currently working on these aspects, in order to overcome these difficulties and to transfer robust industrial methods to our main industrial partners (Airbus and SAFRAN).

- [1] F. Laurin, N. Carrère and JF. Maire, A multiscale progressive failure approach for composite laminates based on thermodynamical viscoelastic and damage models., *Composites Part A: Applied Science and Manufacturing*, 38,198–209,2007
- [2] L. Marcin, JF. Maire, N. Carrère and E. Martin, Development of a Macroscopic Damage Model for Woven Ceramic Matrix Composites *International Journal of Damage Mechanics* 20(6), 939–957, 2010

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# MS10: Innovative trends and applications in fracture mechanics



# Fail-safe Design of Structures Based on the Material Force Approach and Uncertainty Analysis

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In this contribution, a novel approach for a fail-safe design of structures based on an optimization employing a genetic algorithm, which is coupled with structural analysis conducted in the framework of fracture mechanics, and enclosing uncertainty investigations, is proposed. The structural design concepts focus nowadays predominantly on the development of light structures in order to reduce production costs. Thus, each structural component is designed to reach the limit stress state and to use the entire load-bearing capacity. However, if such a structure will be subjected to unforeseen events, e.g. extraordinary loading, impact, fatigue or material defects leading to a failure of a single structural component, the subsequent components will not have enough bearing capacity to carry the additional load. Thus, the resulting reduction in durability of subsequent structural components will induce their sequential failure and, in consequence, a catastrophic failure of the whole structure. In order to prevent systems from the total collapse, fail-safe design approaches are utilized. They assume development of safe structures, which keep their functionality even under particular damage conditions e.g. a local failure of a single structural component.

The optimization of structures with respect to failure mechanisms requires an efficient and a realistic modeling by using a fracture mechanical approach. In this contribution, the investigation of the failure process is introduced within a finite element framework by combination of discrete fracturing and configurational mechanics based criteria. The introduction of a discontinuity field by an r-adaptive node duplication method represents a crack propagation mechanism based on an energy minimization principle for general continua as discussed in [1]. The main feature of this method is to evaluate and to use a general approach to obtain a fracture criterion based on configurational mechanics. In classical mechanics, motion of objects is concerned with the equilibrium depending on mathematical modeling of the solid obeying a variety of constitutive equations and the laws of continuum mechanics. However, in fracture mechanics, the structures cannot be regarded as perfect continuum since they contain defects, voids, dislocations etc. In order to characterize the singularities related to the imperfections in the structure, the concept of a force like quantity in the material space is introduced in parallel to the force acting in physical space, which can be simply interpreted as a local variational of the global energy. As a natural outcome of the energy based formulation, these forces, or rather material forces, arise as crack driving forces in a precracked body and they could be used as a reliable failure and crack path predictor.

In order to cover the whole complexity of the fail-safe design, beside the development of a realistic fracture mechanical model and the implementation of a suitable optimization approach, the analysis of uncertainties is coupled with optimization method. The types of uncertainties in structural design are manifold, e.g. the material properties vary due to unstable production conditions, loads change as a result of diverse structural use or an unexpected loading situation appears, e.g. due to boundary change. All these uncertainties can cause a failure of structural components. In dependency on the uncertainty source and the type of available information: fragmentary, subjective, dubious or statistically well founded and satisfying the i.i.d. paradigm, a suitable uncertainty model is chosen, e.g. fuzziness or probability. Since in most engineering design tasks, polymorphic uncertainty [2] is present, that is uncertainty in form of variability, imprecision and incompleteness appear simultaneously, new modeling approaches are developed. The proposed fail-safe design method includes a sequential coupling of optimization, uncertainty analysis and the numerical deterministic solution. The interaction between the optimization with a genetic algorithm and the material force approach enables to guide the crack path propagation direction into 'safe' structure regions, permitting only local failures and preventing from a total failure of the system. Finally, structural designs can be identified which, if subjected to unforeseen events or uncertainties causing a local failure of components, will sustain the global structure stability.

- [1] K. Özenç; M. Kaliske. An implicit adaptive node-splitting algorithm to assess the failure mechanism of inelastic elastomeric continua. *International Journal for Numerical Methods in Engineering*, 9, 669-688, 2014.
- [2] W. Graf; M. Götz; M. Kaliske. Analysis of dynamical processes under consideration of polymorphic uncertainty. *Structural Safety*, 52 Part B, 194-201, 2015.

# Comparison of fracture prediction models on sheet metal blanking simulations

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Generally in metal forming processes, fracture is synonymous with deficient products, unexpected costs and potential catastrophes, and therefore hopefully prevented during the design stage of a product in order to ensure its integrity and durability. Nevertheless, for a few manufacturing techniques the fracture phenomenon is the driving step of the entire process. This is the case for the sheet metal blanking method where the quality of the final product is directly linked to the resulting shape of the sheared edge. Therefore, the numerical prediction of the failure initiation and further propagation plays an important role in order to ensure the optimal set of process parameters and to fulfill the required product features, especially in mass production.

In the present work, a comparison of existing models to predict the onset of fracture in the numerical simulation of the sheet metal blanking process is carried out. Two sets of different approaches for continuum fracture modeling are considered in this study. On the one hand, a group of different classical fracture criteria, where a failure locus is defined based on the history of internal state variables at each material point without any influence on the constitutive equations (uncoupled approach), is considered [1, 2, 3, 4]. On the other hand, a group of Continuum Damage Mechanics models, where the development of material deterioration is directly included in the constitutive formulation by means of a damage variable, is introduced [5, 6]. Numerical and experimental results are compared in order to evaluate the prediction of the different fracture models considered in this work for various practical cases. The influence of a selected fracture prediction model on some critical process parameters is also studied. In order to take into account the strain-dependent behavior of the material, the large strain extension of the Perzyna's viscoplastic model proposed by Ponthot [7] is considered. Remeshing steps using a novel data transfer method [8] based on finite volumes and numerical integration are utilized to avoid the high distortion of finite elements due to extreme straining in the shearing zone.

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- [1] M. Oyane, T. Sato, Okimoto K., and S. Shima. Criteria for ductile fracture and their applications. *Journal of Mechanical Working Technology*, 4(1):65–81, 1980.
- [2] G. R. Johnson and Cook W. H. Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures. *Engineering Fracture Mechanics*, 21(1):31–48, 1985.
- [3] A. M. Goijaerts, L. E. Govaert, and F. P. T. Baaijens. Prediction of ductile fracture in metal blanking. *Journal of Manufacturing Science and Engineering*, 122(3):476–483, 2000.
- [4] T. Wierzbicki, Y. Bao, Y.-W. Lee, and Y. Bai. Calibration and evaluation of seven fracture models. *International Journal of Mechanical Sciences*, 47(6):719–743, 2005.
- [5] J. Lemaitre. A course on damage mechanics. Springer-Verlag, Berlin Heidelberg New York, 1992.
- [6] L. Xue and T. Wierzbicki. Ductile fracture initiation and propagation modeling using damage plasticity theory. *Engineering Fracture Mechanics*, 75(11):1039–1051, 2008.
- [7] J.-P. Ponthot. Unified stress update algorithms for the numerical simulation of large deformation elasto-plastic and elasto-viscoplastic processes. *International Journal of Plasticity*, 18:91–126, 2002.
- [8] P. Bussetta, R. Boman, and J.-P. Ponthot. Efficient 3D data transfer operators based on numerical integration. *International Journal for Numerical Methods in Engineering*, DOI: 10.1002/nme.4821, 2014.

# Design of Sheet Metal Forming Process for Reducing the Stress Concentration of Corrugated Plate for Molten Carbonate Fuel Cell

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In this study, sheet metal forming processes of the corrugated plates for the molten carbonate fuel cell (MCFC) were numerically investigated by finite element method to obtain optimal design. In the three dimensional forming simulation, the user material subroutine VUMAT was used for simulation of shearing process [1] by applying ductile fracture model in the commercial finite element method software ABAQUS. For more accurate forming simulation, uniaxial tensile test coupled with finite element analysis was used for obtaining real flow stress and determining a critical damage value after onset of necking phenomena [2]. From a number of forming simulations, a new single-stage forming processes that integrates three-stages of shearing, preforming and final forming processes [3] was proposed without local fracture of corrugated plate. Design concept of alleviating locally concentrated stress of slot was drawn from the forming simulations. To verify the new forming process, single-stage forming experiments were conducted. From the above studies, new sheet metal forming process for reducing the stress concentration of slot were successfully demonstrated in the single-stage forming process.

- Somnath Ghosh, Ming Li, Aniruddha Khadke, 3D modeling of shear-slitting process for aluminum alloys, Journal of Materials Processing Technology, 167, 91–102, 2005.
- [2] Seong-Hoon Kang, Young-Seon Lee, Ho Won Lee, Evaluation of flow stress and damage index at large plastic strain by simulating tensile test of Al6061 plates with various grain sizes, International Journal of Mechanical Sciences, 80, 54-61, 2014.
- [3] Dong-Yol Yang, Chang-Whan Lee, Dong-Woo Kang, In-Gab Chang and Tae-Won Lee, Optimization of the preform shape in the three-stage forming process of the shielded slot plate in fuel cell manufacturing. The 11th Int. Conf. on Numerical Methods in Industrial Forming Processes, 1532, 446-451, 2013.

# Analyzing 3D cracks using arbitrary tetrahedral meshes

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In arbitrary tetrahedral meshes, the singularity at the crack front is captured by moving the midside nodes of tetrahedral elements near the crack front to the quarter-point position. This results in the introduction of a nonlinear mapping which reproduces a stress singularity along the crack front regardless of the shape and size of the tetrahedral elements. Using quarter-point tetrahedral elements circumvent the need of a fully structured mesh by pentahedral/hexahedral elements [1] which are difficult to generate, in particularly for complex multi-fracture geometries.

This paper also describes two novel tetrahedral-based methods for the computation of stress intensity factors (SIFs) from the finite element solution. First a direct approach based on the correlation of the displacements over the crack surface. This method is very simple, straightforward, and computationally cheap, which can be readily implemented and computes SIFs approximations with an average error of 2-3%. The second method is based on the domain integral approach [2]. The authors extended the domain integral approach to compute the interaction integral and *J*-integral using unstructured meshes. This is a very reliable and accurate method which computes SIFs with an average error of 1%. Figure 1 shows the variation of the SIFs computed by these two methods along the front of an inclined embedded penny-shaped subjected to uniaxial tension. Overall, these studies demonstrate that arbitrary tetrahedral meshes are reliable, efficient, and accurate for analyzing crack problems in the context of LEFM.

- [1] R.S. Barsoum. On the use of isoparametric finite elements in linear fracture mechanics *International Journal for Numerical Methods in Engineering*, 10, 25-37, 1976.
- [2] J.F. Yau., S.S. Wang., H.T. Corten, A mixed-mode crack analysis of isotropic solids using conservation laws of elasticity ASME Journal of Applied Mechanics, 47, 335-341, 1980.



Figure 1: The variation of normalized mixed mode analytical and numerical SIFs along the front of an inclined embedded penny-shaped crack with the inclination angle of  $\beta = 45^{\circ}$  subjected to uniaxial tension. The results are computed from a displacement correlation approach in (a) and a domain integral method in (b).

# Influence of Autocorrelation Length in Random Lattice-Particle Model

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The main objective of the paper is to describe the role of autocorrelation length of random strength field in lattice-particle model (Cusatis et al. [1, 2]). The original formulation of the lattice-particle model is extended with a random field used for modeling of spatial variability of tensile strength and fracture energy of inter-particle connections [3, 4]. The paper presents lattice-particle models of experimentally tested concrete specimens. The experimental data [5] summarize a series of three point bent concrete specimens of four different sizes with and without notch. The simulations reveal an interesting and clear dependence of the peak load on the autocorrelation length. Apart from the effect of the autocorrelation length on variability of the peak load, we show that the mean value of the peak load of unnotched specimens exhibit a significant decrease when the autocorrelation length is about equal to the size of the fracture process zone.

#### Acknowledgement

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#### Références

- [1] Gianluca Cusatis and Luigi Cedolin. Two-scale study of concrete fracturing behavior. *Eng Fract Mech*, 74(1–2) :3–17, 2007.
- [2] Gianluca Cusatis, Daniele Pelessone, and Andrea Mencarelli. Lattice discrete particle model (LDPM) for failure behavior of concrete. I : Theory. *Cem Concr Comp*, 33(9) :881–890, 2011.
- [3] J. Eliáš, M. Vořechovský, J. Skoček, and Z. Bažant. Stochastic discrete meso-scale simulations of concrete fracture : Comparison to experimental data. *Eng Fract Mech*, 135 :1–16, 2015.
- [4] J. Kaděrová and J. Eliáš. Simulations of bending experiments of concrete beams by stochastic discrete model. In Ferri Aliabadi M.H. Alfaiate J., editor, *Key Engineering Materials*, volume 627, pages 457–460. Trans Tech Publications, 2014.

[5] David Grégoire, Laura B. Rojas-Solano, and Gilles Pijaudier-Cabot. Failure and size effect for notched and unnotched concrete beams. *Int. J. Numer. Anal. Meth. Geomech.*, 37(10):1434–1452, July 2013.

# Investigation into the rupture of the grains in an granular material in an oedometric loading and the fracture energy

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In this work we compared our experimental results with theoretical models which adopts the theory of brittle fracture Griffith (1921), for an oedometric path, and quantify the dissipation during loading and show the evolution of the amount of new fracture surface created during the rupture too, and explain the specific energy of fracture against the mineralogical nature of granular material and explain the contribution of different dissipation during confinement.

We experimentally demonstrate the appearance of the plastic work and also the relationship with the rupture rate, according to the Griffith theory (1921) plastic work is the energy expended in the opening of the crack, so we can justify use this theory for granular soil under oedometric load.

And also the development of new fracture surfaces is shown during a breakage process against its mineralogical types, and we happened to define specific fracture energy using different models based on different approaches.

And it was found that the specific fracture energy constant values during breakage and it depend on the mineralogical nature of the materials.

Key word: fracture, mineralogical, oedometric path, granular materials, dissipation.

#### **References:**

[1]Einav I. (2007c): Soil mechanics: breaking ground. Phil. Trans. R. Soc. A. 365, 2985-3002

[2]Einav I. (2007d): Fracture propagation in brittle granular matter. P. Roy. Soc. A-Math. Phy. 463(2087), 3021-3035

[3]McDowell, G. R., Bolton M. D., Roberston D. (1996).: The fractal crushing of granular materials. J. Mech. Phys. Solids. **44**(12), 2079-2102

[4]Griffith, A.A. (1921): The phenomena of rupture and flow in solids. Phil. Trans. R. Soc. Lond. A 221, 163-198

[5]Russell, A. (2011): A compression line for soils with evolving particle and pore size distributions due to particle crushing. Géotechnique Letters, 1, 5-9
[6]Ovalle, C., Dano, C., Hicher, P.-Y. (2013) "Experimental data highlighting The role of surface fracture energy in quasi-static confined comminution", International Journal of Fracture.

[7]Ovalle, c (2013) Contribution à l'étude de la rupture des grains dans les matériaux granulaires, thèse Doctorat, Ecole Centrale De Nantes .

# **Strain Rate Influence on the Elastoplastic Damage Behaviour of Nodular Cast Iron**

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Due to the high ductility and fatigue strength, the ductile or nodular cast iron is widely used for many structural applications. Numerous investigations have been carried out during the past decades to determine influence of microstructure on the mechanical behaviour of this material under monotonic, cyclic and fatigue loading [1]. However, the literature regarding the dynamic mechanical behaviour of nodular cast iron is limited. This paper presents an experimental and numerical investigation of the dynamic elastoplastic damage behaviour of the nodular cast iron grade EN-GJS-400-18-LT. The experimental procedure includes static and dynamic tensile tests at different strain rates on the flat and cylindrical specimens, as well as three point bending tests. Thereafter, numerical modelling of deformation and failure process of the specimens is conducted by using non-isothermal elastoplastic damage constitutive model and two-dimensional plane stress finite elements [2]. During the experiment the displacement and temperature distribution on the specimens surface is measured by optical measuring system ARAMIS 4M and infrared thermography [1, 3]. This has enabled more precise calibration of material parameters in constitutive relations. In order to validate numerical model, an algorithm has been developed and implemented into the software script to automate the comparison of experimental and numerical data.

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- [1] P. Čanžar, Z. Tonković, et al. Materials Science & Engineering A, 556, 88-99, 2012.
- [2] ABAQUS, Abaqus Software: Version 6.14, 2014.
- [3] M. Vesenjak, L. Krstulović-Opara, et al. Polymer Testing, 32 (8), 1538-1544, 2013.

# Explicit simulation of blade cutting and throughthe-thickness fracture in multi-layer, thin-walled structures

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This work is devoted to a finite element computational tool for the simulation of fracture propagation and blade cutting in multi-layer shell structures.

The thin-walled structure is discretized with eight-nodes solid-shell elements of the type developed by Schwarze and Reese [1] and based on reduced integration with hourglass stabilization. On the one hand, this choice allows for a simple implementation of fully three-dimensional constitutive behaviours and for the cohesive description of fracture phenomena, since solid-shell elements are formulated using displacement degrees of freedom only. On the other hand, the presence of different layers can be accounted for by stacking up one or more solid-shell elements per layer along the thickness.

Through-the-thickness crack propagation is described by means of a cohesive approach. Blade cutting is simulated using directional cohesive elements, as proposed in [4] and in [5], where the transmission of cohesive forces between the two flanks of the crack accounts for the interaction between the blade and the process zone. While the approach in [4] and in [5] was restricted to one-layer thin shells, in the present work crack propagation through the thickness of different layers is explicitly considered.

A conditionally stable explicit time integration is used to deal with the several nonlinearities involved in the problem, such as large deformations, contact, crack propagation and delamination. When a solid-shell discretization is adopted, the stable time step becomes very small, since it is governed by the element thickness, typically small if compared to its in-plane dimensions. This problem is overcome by implementing the selective mass scaling technique proposed in [2-3] for inertia dominated problems and extended to the case of multi-layer structures. According to this approach, the element mass matrix is locally modified to scale down the highest eigenfrequencies with small or negligible changes to the lowest ones. As a result, the stable time step is determined by the minimum in-plane size of the elements only, as for classical shell elements, without accuracy losses.

Benchmarks drawn from the literature on the dynamics fracture of shells are used to test the proposed numerical tool. Moreover, the procedure is applied to the simulation of the cutting of a thin-walled laminate used in the carton packaging industry.

- [1] M. Schwarze and S. Reese. A reduced integration solid-shell finite element based on the EAS and the ANS concept. Large deformation problems. International Journal for Numerical Methods in Engineering, 85, 289-329, 2011.
- [2] G. Cocchetti, M. Pagani and U. Perego. Selective mass scaling and critical time-step estimate for explicit dynamics analyses with solid-shell elements. Computers & Structures, 127:39-52, 2013.
- [3] G. Cocchetti, M. Pagani and U. Perego. Selective mass scaling for distorted solidshell elements in explicit dynamics: optimal scaling factor and stable time step estimate. International Journal for Numerical Methods in Engineering, 101, 700-731, 2015.
- [4] A. Frangi, M. Pagani, U. Perego and R. Borsari. Directional Cohesive Elements for the Simulation of Blade Cutting of Thin Shells. CMES-Computer Modeling in Engineering & Sciences, 7:205-224, 2010.
- [5] M. Pagani and U. Perego. Explicit dynamics simulation of blade cutting of thin elastoplastic shells using "directional" cohesive elements in solid-shell finite element models, Computer Methods in Applied Mechanics and Engineering, 85, 515–541, 2015.

- chapter 10 -

# **66** MS13: Multi-scale analysis

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# X-FEM based computational homogenization-localization for propagating discontinuities

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The aim of this research is to propose a continuous-discontinuous computational homogenization-localization framework to upscale micro-scale localization towards the onset and propagation of a cohesive discontinuity at the macro-scale. A fully coupled micro-macro solution strategy is presented, where the solution procedure for the macroscopic domain is based on the extended finite element method (X-FEM) [1, 2]. The proposed approach departs from classical computational homogenization, which allows to derive the effective stress-strain response before the onset of localization. Upon strain localization, the micro-scale is characterized by a strain localization band where damage grows and by two adjacent unloading bulk regions at each side of the localization zone. To allow for the strain localization band development within the microstructural volume element with minimal interference of the boundary conditions, percolation path aligned boundary conditions have been used, based on the projection of the boundary constraints in the direction of the developing localization band [3]. The micro-scale localization band is lumped into a macroscopic cohesive crack, accommodated through discontinuity enriched macro-scale kinematics. The governing response of the continuum with a discontinuity is obtained numerically based on proper scale transition relations in terms of the traction-separation law and the stress-strain description of the continuous surrounding material at both sides of the discontinuity.

#### Références

- [1] E. Bosco, V.G. Kouznetsova, E.W.C. Coenen, M.G.D. Geers, A. Salvadori, A multiscale framework for localizing microstructures towards the onset of macroscopic discontinuity, *Computational Mechanics*, **54**, 299-319, 2014.
- [2] E. Bosco, V.G. Kouznetsova, M.G.D. Geers, Multi-scale computational homogenization-localization for propagating discontinuities using X-FEM, *International Journal for Numerical Methods in Engineering*, 2014.
- [3] E.W.C. Coenen, V.G. Kouznetsova, M.G.D. Geers, Novel boundary conditions for strain localization analysis in microstructural volume elements, *International Journal for Numerical Methods in Engineering* **90**, 1-21, 2012.

# High toughness fibrillating interfaces in stretchable electronics – a multi-scale numerical and experimental analysis

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Fibrillating interfacial systems, such as large elastic-mismatch elastomer-metal interfaces that are typically used in stretchable electronics applications [1], can exhibit remarkably high values for the interface fracture toughness (i.e.,  $G_c > 1 \text{ kJm}^{-2}$ ) [2]. During peeling of the copper film from the rubber substrate, rubber fibrillation is observed at the copper-rubber interface, as illustrated by the ESEM picture in Fig.1 [3]. The huge gap that exists between the microscopic adhesion energy and the reported macroscopic work-of-separation will be unravelled by means of multi-scale numerical and experimental analysis. This will ultimately lead to guide-lines for engineering interfaces with superior macroscopic toughness values.

This contribution consists of two main parts. First, the contribution of fibril mechanics to the work of separation is quantified by means of a micromechanical model of a single fibril, in which the growth of a nucleated fibril up to the moment of fracture is described [4]. Given the large variation in measured stress-strain curves for rubber materials reported in literature, a small scale single fibril experiment is performed. Furthermore, the fibril fracture stress, which serves as the failure criterion in the model, is extracted from the experiment. The numerical results suggest that for the considered range of material properties, the work of separation is mainly influenced by the fibril fracture stress. In addition, the initial geometry has a profound influence on the obtained work of separation. It is concluded that the work of separation determined from the single fibril model is significantly larger than the intrinsic adhesion energy, yet it remains an order of magnitude smaller than the aforementioned values. Subsequently, a multi-fibril model is proposed to analyse the interaction between the fibrils as well as the interaction between fibrils and the adjacent bulk rubber. It is shown that the discrete nature of the fibrils, the material properties and the sequential failure of fibrils are of key importance in terms of the resulting macroscopic separation energy. A considerable amount of energy is stored in the bulk layer near the fibrils, which is dynamically released upon fibril failure and thereby

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Figure 1: ESEM picture of fibrillation at the copper-rubber interface during peeling [3]

contributes significantly to the work-of-separation. This observation provides a physical explanation for the high macroscopic toughness values, and emphasizes the limits of modelling fibrillating interfaces with a continuum approach.

Second, a multi-scale experimental analysis is presented to explain the characteristic features of the mesoscopic and microscopic delamination front. T-peel and shear-peel delamination tests are conducted with different copper roughness values. Interface characterization at the meso-scale was achieved by matching optical deformation movies to crack propagation simulations (employing a cohesive zone interface model), showing a notably low (macroscopic) mode angle dependency for both roughness types. Real-time in-situ high-resolution ESEM observations revealed that 30  $\mu$ m-long fibrils are formed at the microscopic peel front. Detailed analysis of the fibril shape, distribution and evolution showed that fibril nucleation is caused by mechanical interlocking of the fibril basis in the copper roughness 'valleys', combined with cavitation at the roughness peaks. Postmortem analysis (with a novel quasi-3D global DIC technique) revealed permanent fibril deformation to be negligible.

- [1] Y.Y. Hsu, M. Gonzalez, F. Bossuyt, F. Axisa, J. Vanfleteren, I. de Wolf. The effect of pitch on deformation behavior and the stretching-induced failure of a polymerencapsulated stretchable circuit. *Journal of Micromechanics and Microengineering*, 20(7): 075036, 2010.
- [2] J.P.M. Hoefnagels, J. Neggers, P.H.M. Timmermans, O. van der Sluis, M.G.D. Geers. Copper-rubber interface delamination in stretchable electronics. *Scripta Materialia*, 63(8):875-878, 2010.
- [3] O. van der Sluis, Y.Y. Hsu, P.H.M. Timmermans, M. Gonzalez, J.P.M. Hoefnagels. Stretching induced interconnect delamination in stretchable electronic circuits. *Journal of Physics D: Applied Physics*, 44:034008, 2011.
- [4] B.G. Vossen, P.J.G. Schreurs, O. van der Sluis, M.G.D. Geers. Multi-scale modeling of delamination through fibrillation. *Journal of the Mechanics and Physics of Solids*, 66: 117Ű132, 2014.

# Compelling issues in modelling concrete with recycled aggregates

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The use of recycled aggregates (RA) is very attractive from the point of view of reduction of concrete production costs. This kind of aggregates typically consists in steel production by-products or products of previous demolitions.

An extensive scientific research has been done on this subject over the last decades, namely concerning how the use of RA might influence the performance of concrete for different aims, not merely structural (e.g. radioactive shielding for nuclear purposes) [1,2]. The scope of this work is to illustrate the challenging aspects of RA modelling in concrete mixtures and formalize a procedure to cope with them.

To catch the behaviour at the mesoscale of such a composite material in terms of thermo-hydro-mechanical response [3] some compelling issues need to be addressed while modelling, first of all to give an accurate reproduction of the geometry of these aggregates, which is known to be quite complex. At this purpose the 3D laser-scanning technique has been adopted to acquire with fairly good approximation the external irregular surface of some sample aggregates coming from previous demolition, which are characterized by a natural aggregate (NA) partially surrounded by a variable amount of old mortar (M). Starting from the external surface the whole inner volume has been reasonably distinguished into NA or M, based on continuity requirements for the splines describing the external surfaces, in order to discretize it in space and carry out FEM analyses on a concrete sample made of such a kind of aggregates.



Fig.1: Configuration of a typical RA from previous demolition a), 3D laser scanner output b), graphical elaboration into surfaces (tessellation) c), 3D geometrical FEM model d).

Secondarily, a specific mechanical constitutive behaviour must be carefully calibrated for each phase of the composite: concrete paste, natural aggregate, mortar, interfacial transition zone between cement and aggregate. A specific damage-plasticity formulation has been developed in this sense to reproduce the softening behaviour encountered in prismatic specimens made with recycled aggregates of this kind under uniaxial compression tests.

- R.V. Silva, J. de Brito, R.K. Dhir, Prediction of the shrinkage behavior of recycled aggregate concrete: a review, Construction and Building Materials, 77, 327–339, 2015.
- [2] C. Pellegrino, P. Cavagnis, F. Faleschini, K. Brunelli, Properties of concretes with Black/Oxidizing Electric Arc Furnace slag aggregate, Cement & Concrete Composites, 37, 232-240, 2013.
- [3] G. Xotta, V.A. Salomoni, C.E. Majorana, Thermo-hygro-mechanical meso-scale analysis of concrete as a viscoelastic-damaged material, Engineering Computations, 30(5), 728-750, 2013.

## Multiscale Schemes for Modelling Fracture in Soft and Brittle Materials under Impact

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A computational homogenization scheme is presented to model heterogeneous materials under impact loading for brittle and soft materials. In the computational homogenization method the overall properties at the macro-scale can be obtained by solving a boundary value problem for a representative volume element (RVE) assigned to each material point of the macro-scale model. Under impact loading, the wave length of propagating waves may be comparable to the size of the RVE and wave reflections at local-scale material interfaces then cause wave dispersion. In order to model dispersion effects using a computational homogenization scheme, one has to account for the inertia forces at the local-scale. However, performing a dynamic analysis at the local-scale results in spurious wave reflections at the RVE boundaries.

In order to avoid these wave reflections at the RVE boundaries, the local-scale model is solved as a quasi-static problem while a dynamic analysis is performed for the global-scale model [1]. Dispersion effects in the bulk material caused by reflection of the wave at the local-scale material interfaces are captured by accounting for the inertia forces at the local-scale model via a so-called dispersion tensor which depends on the heterogeneity of the RVE [1, 2]. When strain localization occurs, a continuous-discontinuous computational homogenization method [1] is employed to obtain the traction-separation law for macro-cracks using averaged properties calculated over the damaged zone in the RVE. An updated Lagrangian formulation is used at both global-scale and local-scale to account for large deformations/large strains for soft materials. Numerical examples for various loading frequencies, loading rates and hard/soft materials are given and the multi-scale model results are compared to direct numerical simulation results which show a good agreement. The objectivity of the multi-scale scheme with respect to the RVE size is also examined.

- [1] A. Karamnejad and L. J. Sluys: A dispersive multi-scale crack model for quasi-brittle heterogeneous materials under impact loading. *Computer Methods in Applied Mechanics and Engineering*, 278, 423-444, 2014.
- [2] J. Fish, V. Filonova and S. Kuznetsov: Micro-inertia effects in nonlinear heterogeneous media *International Journal for Numerical Methods in Engineering*, 91, 1406-1426, 2012.

# **Computational homogenization of fracturing** continua using weakly periodic boundary conditions

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Computational homogenization of elastic media with propagating cracks is studied, whereby crack propagation is modeled with the eXtended Finite Element Method (XFEM) and the concept of material forces. More precisely, the aim of this work is to study computational homogenization of a material with propagating cracks prior to localization. To this end, the effective stress and strain in the microstructure are computed by homogenizing the response of a Statistical Volume Element (SVE). When choosing suitable Boundary Conditions (BCs) on the SVE, it is important to notice that classical choices (Neumann, Dirichlet and strong periodic) are problematic if cracks intersect the SVE boundary. To overcome the deficiencies of classical boundary conditions and allow for non-periodic meshes, we employ a weak format of micro-periodicity based on the work by Larsson et al. [1].

We consider homogenization of an SVE occupying the domain  $\Omega_{\Box}$  with the external boundary  $\Gamma_{\Box}$  and internal boundaries representing micro-cracks that may propagate. Following Larsson et al. [1], we may evaluate the response of a single SVE by solving the following problem:

Find  $\boldsymbol{u} \in \mathbb{U}_{\Box}$  and  $\boldsymbol{t} \in \mathbb{T}_{\Box}$  such that

$$\frac{1}{|\Omega_{\Box}|} \int_{\Omega_{\Box}} \boldsymbol{\sigma} : \boldsymbol{\epsilon} \left[ \delta \boldsymbol{u} \right] dV - \frac{1}{|\Omega_{\Box}|} \int_{\Gamma_{\Box}} \boldsymbol{t} \cdot \delta \boldsymbol{u} dS = 0 \qquad \forall \delta \boldsymbol{u} \in \mathbb{U}_{\Box}, \\ -\frac{1}{|\Omega_{\Box}|} \int_{\Gamma_{\Box}} \delta \boldsymbol{t} \cdot \boldsymbol{u} dS = -\frac{1}{|\Omega_{\Box}|} \int_{\Gamma_{\Box}} \delta \boldsymbol{t} \cdot \bar{\boldsymbol{\epsilon}} dS \qquad \forall \delta \boldsymbol{t} \in \mathbb{T}_{\Box},$$
(1)

where  $\boldsymbol{\sigma}$  is the Cauchy stress,  $\boldsymbol{\epsilon} \left[ \boldsymbol{u} \right] \stackrel{\text{def}}{=} \left( \boldsymbol{u} \otimes \boldsymbol{\nabla} \right)^{\text{sym}}$  is the small strain tensor and  $\boldsymbol{t}$ is the traction on the boundary. In standard fashion, we may then determine the relation between the effective stress and strain, given by  $\bar{\boldsymbol{\sigma}} = \frac{1}{|\Omega_{\Box}|} \int_{\Omega_{\Box}} \boldsymbol{\sigma} \, \mathrm{d}V$  and  $\bar{\boldsymbol{\epsilon}} = \frac{1}{|\Omega_{\Box}|} \int_{\Gamma_{\Box}} \boldsymbol{u} \otimes \boldsymbol{n} \, \mathrm{d}S, \text{ respectively.}$ The weak form of the SVE problem above is solved using an XFEM approxi-

mation to account for displacement discontinuities. Following standard procedures

(see e.g. the review in [2]), Heaviside enrichment is used in elements completely cut by a crack whereas elements containing a crack tip are enriched with asymptotic functions. To facilitate modeling of propagating cracks, we combine XFEM with the concept of material forces, see e.g. [3].

Combining weakly periodic boundary conditions with XFEM and the concept of material forces allows crack propagation in the microstructure to be studied, see Figure 1 for an example showing the response of a single SVE. By varying the traction approximation, different boundary conditions ranging from Neumann to strong periodic can be realized. Using this framework, we aim to identify a suitable boundary condition on the SVE in the presence of propagating cracks. Preliminary results indicate that a piecewise constant traction approximation, with traction discontinuities where cracks intersect the SVE boundary, is effective in terms of convergence with increasing SVE size. In addition to these preliminary studies, further investigations of the potential of using these boundary conditions for the modeling of fracturing microstructures have been conducted. The results of these investigations will be presented at the conference.



Figure 1 – Microstructure simulation example.

- F. Larsson, K Runesson, S. Saroukhani and R. Vafadari. Computational homogenization based on a weak format of micro-periodicity for RVE-problems. *Computer Methods in Applied Mechanics and Engineering*, 200, 11-26, 2011.
- [2] T.P. Fries and T. Belytschko. The extended/generalized finite element method: an overview of the method and its applications. *International Journal for Numerical Methods in Engineering*, 84(3), 253-304, 2010.
- [3] P. Steinmann, D. Ackermann and F.J. Barth. Application of material forces to hyperelastostatic fracture mechanics. II. Computational setting. *International Journal of Solids and Structures*, 38, 32-33, pp. 5509-5526, 2001.

# **3D dislocation dynamics simulation of crack** shielding and blunting in FCC metals

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Subjected to low-amplitude cyclic loading, ductile crystalline solids like FCC metals undergo plastic strain localisations, which may lead to the formation of cracks and subsequent fracture. Experimental evidence shows that crack growth and propagation at low stress intensity strongly depends on the microstructural behavior and characteristics of the material. Among the mechanisms involved, the interactions of the dislocations with the stress concentration ahead of the crack-tip (plastic zone) seem to play a decisive role. Two main phenomena are of interrest :

- The shielding effect due to the increase or decrease of the stress acting on the crack surfaces due to the dislocation microstructure developing around the crack tip.
- The blunting effect induced by the emmision or absorption of dislocation loops at the crack tip

Modeling these phenomena is a long-standing and complex problem. So far, existing models are essentially 2D [1, 2, 3] and only few attempts in 3D have been made [4, 5]. 3D dislocation dynamics (DD) simulations are then needed to quantitatively investigate the plastic deformation restraining crack propagation through shielding and blunting mechanisms. Only in 3D the influence of crystal symmetry, exact slip system activity and thermally activated processes like dislocation cross-slip can be precisely taken into account.

To model this complex boundary value problem, the Discrete-Continuous Model (DCM) [6] is used to reproduce the interactions of realistic dislocation microstructures with a short sharp initial crack. Several crack orientations are studied in a Cu monocrystal. Analyses of the respective slip system activity in the plastic zone and the evolution of the shielding and blunting mechanisms at the crack tip are presented. These results are of particular interest for the development of dislocation density based models of crystal plasticity devoted to the complex problem of crack growth in fatigue. To evaluate the strain energy release around the crack tip and to quantify the fracture energy for the crack to propagate, a G-theta integral method is tested on the MDC calculations [7].

Results using the MDC methodology are then compared to Crystal Plasticity Finite Element (CPFEM) simulations using a constitutive laws [8] we optimized to model Cu Single crystal plasticity. Strengths and weaknesses of both approaches are discussed.

- [1] W. A. Curtin, V. S. Deshpande, A. Needleman, E. Van der Giessen, and M. Wallin. *International Journal of Fatigue*, (9):1511–1520, Sep.
- [2] D. S. Balint, V. S. Deshpande, A. Needleman, and E. Van Der Giessen. *Philosophical Magazine*, (26-27):3047–3071, Sep.
- [3] S. G. Roberts, A. S. Booth, and P. B. Hirsch. *Materials Science and Engineering: A*, (1–2):91–98, 3.
- [4] B. Devincre and S. G. Roberts. Acta Materialia, (7):2891–2900, Jul.
- [5] C. Déprés, M. Fivel, and L. Tabourot. Scripta Materialia, (12):1086–1089, 6.
- [6] A. Vattré, B. Devincre, F. Feyel, R. Gatti, S. Groh, O. Jamond, and A. Roos. *Journal* of the Mechanics and Physics of Solids, (0):491–505, 2.
- [7] Ph Destuynder, M Djaoua, and S Lescure. *Journal de mécanique théorique et appliquée*, (1):113–135.
- [8] L. Méric, P. Poubanne, and G. Cailletaud. *Journal of Engineering Materials and Technology*, (1):162–170, 01.

# Numerical simulation of damage patterns in brittle materials under thermo-mechanical loading

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Ceramic materials, for example refractories, initially contain a multitude of defects such as voids, microcracks, grain boundaries etc. Particularly being exposed to high temperatures and loaded by thermal shocks, the macroscopic properties such as effective compliance, strength and lifetime are essentially determined by microscopic features of the material. The deformation process and failure mechanisms are going along with the creation of new microdefects as well as their growth and coalescence. A brittle damage model based on multiscale considerations and homogenisation procedures is presented. Simple defects like idealized straight cracks or statistically distributed cracks with different orientations are homogenized analytically and integrated into a finite element (FE) framework. More sophisticated models including e.g. grain boundaries have to be implemented in representative volume elements (RVE) and treated numerically. For linear reversible processes the effective properties, e.g. effective elastic constants, may be determined from RVE simulations being decoupled from the global boundary value problem. If damage or other irreversible processes are involved, a fully coupled multiscale FEMsimulation is inevitable. In order to properly model the thermo-mechanical coupling, the temperature-dependence of material constants is taken into account. Moreover, fracture and damage mechanical approaches are combined based on a single approach. Thus, interactions of macroscopic crack tips and microstructural features can be taken into account. Numerical examples reveal fatigue damage patterns under repeated thermal shocks finally leading to a failure of the structure.

# Multiscale modelling of propagating fracture in quasi-brittle materials: a continuum approach

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The work presents an approach to computational multiscale modelling of material failure using finite element analysis at two material scales ( $FE^2$ ). Its goal is beyond the simple computational homogenization, and it aims at inserting the resulting, non-smooth, homogenized constitutive model into a computational scheme for modelling the onset and propagation of the material failure at the structural macro-scale. In this context, the main features of the approach are the following:

- Extends the homogenization paradigms for smooth problems —typically the Hill-Mandel principle and the stress/strain homogenization procedures— to non-smooth problems, with no fundamental changes.
- In both scales, a continuum (stress-strain) constitutive relationship is considered, instead of the most common discrete traction/separation-law, this contributing to provide a unified setting for smooth and non-smooth problems. This is achieved by resorting to the Continuum Strong Discontinuity Approach (CSDA) to material failure [1].
- As for the multiscale modelling issue, it involves a crucial additional entity: an internal (or characteristic) length, which is point wise obtained from the geometrical features of the failure mechanism developed at the lower scale. As a specific feature of the presented approach, for the non-smooth case this internal length is exported, in addition to the homogenized stresses and the tangent constitutive operator, to the macro-scale, and considered the bandwidth of a propagating strain localization band, at that scale.
- Consistently with this internal length, a specific computational procedure, based on the crack-path-field and strain injection techniques, recently developed by the authors [2] is then used for modelling the onset and propagation of this localization band, at the macro-scale.

Representative simulations show that the resulting approach provides mesh objective results with respect to, both, size and bias of the upper-scale mesh, and with respect to the size of the lower-scale RVE/failure cell. The continuum character of the approach confers to the formulation a minimally invasive character, with respect to standard procedures for computational one-scale homogenization and modelling of propagating material failure. The issue of reducing the computational cost using High Performance Reduced Order Modelling (HP-ROM) techniques is also considered.

- Oliver, J. and A.E. Huespe, "Theoretical and computational issues in modelling material failure in strong discontinuity scenarios". *Comp. Meth. Appl. Mech. Engng*, 193, 2987-3014 (2004).
- [2] Oliver, J., I.F. Dias, and A.E. Huespe, "Crack-path field and strain-injection techniques in computational modeling of propagating material failure". *Comp. Meth. Appl. Mech. Engng*, **274**, 289-348 (2014).

#### **3D Fracture Simulation in Porous Graphite**

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Synthetic graphites have a system of pores with spatially complex shapes. There are various types of industrial synthetic graphites; for example Gilsocarbon forms the moderator in the UK nuclear Advanced Gas Reactors (AGR) and is used here as a model material. Gilsocarbon consists of filler particles with millimeter size and a matrix that comprises pitch binder and ground filler particles; it has a characteristic pore size distribution ranging from a few nanometres to several millimetres. The total porosity of Gilsocarbon in the as-manufactured or "virgin" state is appoximately 20%, and is changed gradually during reactor operation by the combination of fast neutron irradiation and radiolytic oxidation. The porous structure of a brittle material, such as graphite, greatly influences its strength and fracture behaviour, and a microstructure-sensitive model, reflecting the key parameters of the real microstructure, is necessary to fully evaluate and predict the effects of the evolving porosity. The considerable range of the pore sizes in synthetic graphites requires the creation of a model over different length scales, involving several modeling techniques simultaneously.

An original method of elastic stress calculation has been developed based on the cellular automata (CA) technique, which allows the efficient estimation of the stress concentrations in a material with any 3D shapes of pores. The method considers the cells corresponding to the pores and solid as local elements with different properties; the cells corresponding to the solid matrix are denoted as inactive elements of the structure, and the cells of pores are active elements. The stress field generated by each active element is computed using elasticity theory for the stress around a void; linear elastic fields have been assumed at this stage, but the model is applicable to non-linear elastic and elastic-plastic fields. The total stress field in the porous structure is obtained as a superposition of elastic stress fields generated by all cells with the active state. Periodic boundary conditions were adopted in a 3D model. The approach assumes that configuration of inactive elements in the CA lattice can be random; this allows the calculation of the stress distribution around the pores with arbitrary shapes. The results have been calibrated with the analytical solution for an infinite volume with a central spherical void; the stress concentration factors obtained for non-spherical pores have been validated against the analytical solution for the stresses at the boundary of an ellipsoidal cavity of arbitrary aspect ratio [1]. The CA model is shown to be sufficiently accurate in comparison with a solid finite element model of the same resolution, and is very computationally efficient, being at least one order faster than a finite element model of the same level of discretisation.

This efficiency is required for the simulation of the effects of microstructure on fracture initiation and propagation, which is done using a CA fracture algorithm. Any

arbitrary criterion of crack initiation can be applied after the calculation of the stress value in each cell of the model; those cells falling under this criterion are marked as cracked. Recalculation of the stress distribution, taking into account the cracked cells as active cells, is then performed for the damaged microstructure.

The CA model has been applied to a microstructure description obtained from 3D computed X-ray tomography images of virgin Gilsocarbon, collected at Diamond Light Source (experiment EE8519) with a resolution of 1.8  $\mu$ m/voxel. The properties of the matrix and filler particles were both simulated, using segmented images of the microstructures to describe their porosity; fracture of the simulated volumes was then modelled under uniaxial loading using a critical strain energy release rate criterion. For comparison, synthetic microstructures were created with pores of different shapes, including spherical, ellipsoidal, and arbitrary shapes, to assess their relative effects on material strength. The effects of progressive growth of pores, approximating radiolytic oxidation, was also examined.

This analysis demonstrates the utility of the CA model, which is here used as an independent technique to simulate the strength of porous microstructures, finding that that failure stress distribution becomes less scattered as the mean strength value decreases with increasing porosity. The CA model can be combined with multi-scale methods to simulate fracture nucleation and propagation by providing the stochastic properties of the components of a heterogeneous microstructure for a Cellular Automata Finite Element Method [2] that simulates the failure of test specimens and components of different geometries.

- [1] M.A. Sadowski, E. Sternberg. Stress Concentration Around a Triaxial Ellipsoidal Cavity. Journal of Applied Mechanics, 16, 149-157, 1949.
- [2] L. Saucedo-Mora, T.J. Marrow. 3D Cellular Automata Finite Element Method with Explicit Microstructure: Modeling Quasi-brittle Fracture using Meshfree Damage Propagation. In Procedia Materials Science, 3, 1143–1148, 2014.

# Crystal plasticity model for describing fatigue of a lead-free solder under passive temperature cycling

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Solder joints used in power modules for electric vehicles are subjected to severe stresses caused by thermal cycling. The latter occurs in the course of environmental temperature changes (passive cycling) and/or power variations (active cycling). The resulting lifetime of the solder material is thus largely function of the level of these stresses. In general, the solder layer possesses a very small thickness on the order of a few grains, resulting in a critical dependence of the system performance on the micromechanical behavior of the solder joint. Parameters such as grain size, crystallographic orientations or grain boundary strength play a significant role in the overall response of the system. This suggests considering a multi-scale approach to describe accurately the microscopic mechanisms related to deformation and failure of the material. In this study, a finite element model, based on crystal plasticity as developed in [1], is used to study the fatigue of a solder joint integrated in a complete IGBT (Insulated-Gate Bipolar Transistor) module.

Due to the heavy computational cost that the multi-scale approach implies, the submodeling technique is employed to model only the zone of interest while accounting for the exact boundary conditions. To do this, a macroscopic simulation of the whole package is first carried out; then the zone of interest is extracted and meshed finely so that its microstructure is explicitly reproduced. In the global simulation of the whole system, the viscoplastic model of Anand is used for the solder material and the IGBT

package is subjected to passive thermal cyclic loading. The finite element results are analyzed in order to obtain not only a first evaluation of the complete joint response, but also the boundary conditions, in the stabilized cycle, for the driven nodes of the submodel. In the local simulation, these driven nodes, matching the results from the global model, allows to compute more precisely the submodel, which is reduced assembly taken as а



Strain-Stress hysteresis of solder in 10 cycles

consisting only of the chip, the solder and the substrate. In this submodel, the critical zone of the solder material is meshed at the level of grain microstructure. The needed crystal plasticity parameters were identified with the help of the Berveiller-Zaoui homogenization scheme [2] using experimental macroscopic data. Finally, the accumulated plastic slip in the solder material is estimated and used in a low-cycle fatigue law of Coffin-Manson type [3] to predict the number of cycles to failure for the given loading.



- [1] Y. Huang. A user-material subroutine incorporating single crystal plasticity in the abaqus finite element program. Division of Applied Sciences, Harvard University, Cambridge, MA, 1991.
- [2] M. Berveiller et A. Zaoui. An extension of the self-consistent scheme to plastically flowing polycrystal. J. Mech. Phys. Sol., 26:325-344, 1979.
- [3] F. Dunne, A. Wilkinson, R. Allen. Experimental and computational studies of low cycle fatigue crack nucleation in a polycrystal. Int. J. Plasticity 23 (2), 273-295, 2007.

## **Computational modeling of molecular discrete systems: the role of nonlocality on flaw-tolerance**

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Long-range interactions can be investigated at the micro- and nano-scales according to discrete models, like the lattice beam model by van Mier [1] and coworkers, or the Born models [2], able to catch phenomena for which continuum descriptions break down. Nowadays, the high computational power and development of open source software make possible simulations of wide scalable systems, in which the laws of motion and the mutual interactions among up to million particles (or molecules) are governed by highly nonlinear interatomic potentials. This is the case of Molecular Dynamics (MD), in which a large set of differential equations are solved using explicit time integration techniques with a reduced computational cost.

In this work, it is proposed an evaluation of nonlocality effects related to molecular interactions within a regular periodic hexagonal 2D discrete system with or without flaws, comparing two different computational approaches applied to the same geometric scheme.

The former is a displacement-based finite element (FE) formulation [3] for large displacements, in which the force field used to take into account both bonded (local, chemical) and non-bonded (nonlocal, physical) contributions to the system energy is based on a modified Lennard-Jones potential in which both compressive and tensile fields are not limited by a cut-off interaction distance. Links are represented by nonlinear spring elements, whose stiffness is conditioned not only by the nearest neighboring atoms but also by the subsequent atoms or rather parts of molecules. Definitely, the nonlocality can be quantified through a dimensionless index, expressed as the ratio between the length of a link joining two neighboring atoms and an hypothetical imposed cut-off radius at which pairing mutual interactions among atoms almost vanish. Allowing an increase of this index corresponds to consider an interatomic force vs. distance relation with a smooth decay far from the origin leading to a wide redistribution of forces inside the system.

The latter is an evolution of the previous model according to the theory proposed in [4], in which a 4-nodes elements was defined to simulate the interactions among atoms governed by a DREIDING force field [5]. The original simulation is then extended to dynamics and implemented in the finite element analysis programme FEAP [6] by the

present authors. In this approach, the same sample is modeled according to a pure Molecular Dynamics (MD) formulation numerically solved in the software LAMMPS [7]. The considered systems of molecules are subjected to a uniaxial tensile test and their response based on the previous proposed models are simulated depending on the considered range of nonlocal interaction of the system. For both of them, the capacity of the system to tolerate flaws is evaluated by examining the statistical redistribution of forces in the underlying network, in order to understand the features of the redistribution mechanisms occurring in nonlocal discrete systems in the presence of defects.

- J.G.M. van Mier, E. Schlangen, A. Vervuurt. Lattice type fracture models for concrete, H.B. Mühlhaus (Ed.), Continuum Models for Materials with Microstructure, John Wiley & Sons, 341-377, 1995.
- [2] A.V. Marenich, C.J. Cramer, D.G. Truhlar. Generalized Born solvation model SM12. Journal of Chemical Theory and Computation, 9, 609-620, 2013.
- [3] A. Infuso, M. Paggi, Flaw-tolerance of nonlocal discrete systems and interpretation according to network theory, Fracture and Structural Integrity, 29, 302-312, 2014.
- [4] L. Nasdala, A. Kempe, R. Rolfes. The molecular dynamic finite element method (MDFEM), Computer, Materials & Continua, 19, 57-104, 2010.
- [5] S.L. Mayo, B.D. Olafson, W.A. Godard III. DREIDING: A generic force field for Molecular Simulations, Journal of Physical Chemistry, 94, 8897-8909, 1990.
- [6] O.C. Zienkiewicz, R.L. Taylor. The Finite Element Method, 6th edition, Elsevier Oxford, Volumes 1 and 2, 2005.
- [7] S. J. Plimpton, Fast Parallel Algorithms for Short-Range Molecular Dynamics, Journal of Computations in Physics, 117, 1-19, 1995. (lammps.sandia.gov).

# A Concurrent Parallel Multiscale Algorithm for Large 3D Continuum/Atomistic Simulations with Applications to Dislocations

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Deformation and fracture processes in engineering materials often require simultaneous descriptions over a range of length and time scales, with each scale using a different computational technique. Here we present a high-performance parallel 3D computing framework for executing large concurrent multiscale studies that couple an atomic domain, modeled using molecular dynamics, and a continuum domain, modeled using explicit finite elements. The coupling is achieved with the robust Coupled Atomistic/Discrete-Dislocation (CADD) method. The key features here are the extension to 3D and, moreover, an implementation within the parallel molecular dynamics code LAMMPS that enables use of all the tools associated with this popular open-source code. As an example, we show the application of our multiscale method by discussing robust results obtained from an atomic scale analysis of the dislocation line tension with two distinct interatomic potentials and demonstrating an effective Peierls Stress applicable to curved dislocation structures. Our multiscale method allows us to reproduce the results of extremely large atomistic simulations at a much lower computational cost, thus enables us to remove the finite the size effects normally influencing small finite-size atomistic simulations.

# Determination of the Effective Fracture Behavior of Heterogeneous Materials by a Phase Field Approach

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The study of fracture in heterogeneous composite materials is a difficult topic since macroscopic failure is usually accompanied by various fracture events on the microstructural level. Cracks do not necessarily propagate continuously but may be arrested, deflected or even bifurcated at obstacles like material interfaces. Crack kinking, branching, and the nucleation of new micro cracks lead to complicated fracture patterns. In this work, we study the fracture behavior of composite materials by means of a phase field model for fracture which is based on the regularized variational formulation of brittle fracture from [1]. The phase field approach is especially suitable in this context, because it provides a holistic description of fracture that is able to reproduce all of the mentioned phenomena and allows a comparably convenient discretization with finite elements. A similar fracture model was used in [2] in an effort to determine the homogenized effective fracture resistance of composites. The proposed definition of the effective fracture resistance is based on the evaluation of the far field  $\mathcal{I}$ -integral while a crack propagates through the composite. In the present work we review this approach using the concept of configurational forces within the phase field fracture model. This ansatz enhances the understanding of the effects of the microstructure on the far field *I*-integral because it provides an intuitive visualization of the different contributions to this anticipated macroscopic quantity. Regarding the determination of the effective fracture resistance, also the interplay of the far field and the near tip *I*-integral is of interest, see [3]. Given a proper separation of scales between the phase field regularization length and the structural dimensions of the composite, the configurational forces in the vicinity of the crack tip provide insight into the local driving mechanisms of crack propagation, i.e. the near tip J-integral. The analysis of these local crack driving forces is relevant to understand crack arrest at material interfaces. It is found that this crack arrest and subsequent re-nucleation has a crucial impact on the observed far field  $\mathcal{I}$ -integral. Thus, the crack nucleation stress which is directly linked to the regularization parameter of the phase field model (see e.g. [4]) plays an important role.

- [1] B. Bourdin, G. Francfort and J.-J. Marigo. The Variational Approach to Fracture. *Journal of Elasticity*, 91(1), 5-148, 2008.
- [2] M.Z. Hossain, C.-J. Hsueh, B. Bourdin and K. Bhattacharya. Effective toughness of heterogeneous media. *Journal of the Mechanics and Physics of Solids*, 71, 15-32, 2014.
- [3] F.D. Fischer, J. Predan, R. Müller and O. Kolednik. On problems with the determination of the fracture resistance for materials with spatial variations of the Young's modulus. *International Journal of Fracture*, 190(1), 23-38, 2014.
- [4] C. Kuhn and R. Müller. Interpretation of parameters in phase field models for fracture. *Proceedings in Applied Mathematics and Mechanics*, 12(1), 161-162, 2012.

## Fracture initiation in multi-phase materials: a micromechanical approach

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Multi-phase materials are used in many modern engineering applications. For instance in the automotive industry many structural components are manufactured using dual-phase steel. In such applications a compromise is made between strength and ductility to maximize the structural integrity of the car body while minimizing the weight, but at the same time ensuring formability. Multi-phase materials provide both strength and ductility by combining two phases with distinct mechanical properties at the level of the microstructure (i.e. the level of individual grain in dual phase steel). Generally these materials consist of a comparatively hard but brittle inclusion phase embedded in a soft but ductile matrix.

The mechanical response (elasticity and plastic hardening) is reasonably well understood, and can be predicted using models with varying degrees of complexity ranging from the analytical rule of mixtures to numerical models. However the failure of such materials is less understood. Several authors have studied failure of multi-phase materials (for example [2, 4, 6]), but very few perform a systematic study. Kumar et al. [4] use a statistical reconstruction of the microstructure to identify the critical configuration for the onset of fracture. In addition, De Geus et al. [1] identify the average microstructure around the onset of fracture of the soft and ductile matrix phase.

In the current article the initiation of ductile fracture of the soft phase and brittle fracture of the hard phase is considered at the same time, using a numerical model. Damage indicators for both phases are used that are taken from the literature and are based on experiments (see [5]). The competition between these two fracture initiation mechanisms is studied for different macroscopic stress states characterized by the triaxiality. An idealized microstructural model is used in the form of a volume element in which the hard and soft phase are randomly distributed. A large ensemble of these volume elements is used to limit the effect of statistical fluctuations. The initiation of macroscopic fracture is determined from this ensemble, rather than from a single microstructural volume element.

It is observed that the strain at which macroscopic fracture initiates decreases with increasing applied triaxiality. In contrast to common ductile materials however a rapid decrease of fracture strain is observed above a certain critical value of triaxiality. By considering the damage indicators of the phases separately it is found that this rapid decrease in fracture strain is caused by a transition from fracture initiation dominated by the soft

phase at lower triaxialities to fracture initiation dominated by the hard phase at high triaxialities. A similar observation is made experimentally by Hoefnagels et al. [3]. In this article, the competition is studied for a large range of phase contrast and volume fractions.

The average microstructure around the initiation of fracture is calculated at different values of triaxiality. Figure 1 shows the average phase around fracture initiation in the center (a) for a low triaxiality at which the fracture initiation is dominated by the soft phase and (b) for a high triaxiality at which it is dominated by the hard phase. A negative value may be interpreted as soft phase, a positive as hard phase, while a zero value corresponds to no preferential phase. The results show a remarkable similarity between both fracture initiation mechanisms. The average distribution comprises a band of hard phase in the direction of tensile strain is interrupted by bands of soft phase in the direction of shear.



Figure 1: The average microstructure around the initiation of the fracture (a) when only soft phase damage is considered and (b) when only hard phase damage is considered. Negative values (gray) may be interpreted as on average soft phase, positive values (red) as on average hard phase. Pure shear is applied using a tensile strain in horizontal direction and a compressive strain in vertical direction.

- [1] T. W. J. de Geus, R. H. J. Peerlings, M. G. D. Geers. Submitted for publication, 2014.
- [2] H. Ghadbeigi, C. Pinna, S. Celotto, and J. R. Yates. Mater. Sci. Eng. A, 527(18-19):5026–5032, 2010.
- [3] J. P. M. Hoefnagels, C. C. Tasan, F. J. Peters, F. Maresca, V. G. Kouznetsova. *Submitted for publication*, 2014.
- [4] H. Kumar, C. L. Briant, and W. A. Curtin. Mech. Mater., 38(8-10):818-832, 2006.
- [5] N. Vajragupta, V. Uthaisangsuk, B. Schmaling, S. Münstermann, A. Hartmaier, and W. Bleck. Comput. Mater. Sci., 54:271–279, 2012.
- [6] J. J. Williams, Z. Flom, A. A. Amell, N. Chawla, X. Xiao, and F. De Carlo. Acta Mater., 58(18):6194– 6205, 2010.

## Compressive failure of composites: A computational homogenization approach

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In nonlinear computational homogenization, two continuous models are nested, which permits to introduce constitutive laws only at the local level. We discuss these

double scale modeling's especially when there is a bifurcation at the local level. It has been early established [1] that a local bifurcation is strongly connected to a macroscopic loss of ellipticity and therefore, leads to an illposed problem. Such double scale models have been solved by finite element discretizations at both levels (the so called  $FE^2$  method); see Fig. 1.

This concurrent model has been



Fig. 1: Computational homogenization: two nested models with constitutive law at the local level

applied for long fiber composites [2] by introducing a basic cell whose length is an assumed known microbuckling wavelength. The corresponding instability depends on micro quantities such as fiber waviness or matrix nonlinear behavior, but also of on macro quantities such as plate thickness or loading type. The FE<sup>2</sup> method permits to analyze these double scale instabilities in a coupled and concurrent manner.

The loss of ellipticity at the macroscopic level yields an obvious failure criterion that has been compared with the shear band approach of [3]. The two approaches lead to very close failure predictions, but contrarily to the shear band approach, the double scale ellipticity requires a single constitutive assumption.

Nevertheless, the model fails after this macroscopic loss of ellipticity, where the response becomes mesh dependent. Hence, a generalized continuum approach is necessary to predict the behavior close to the failure. Various possibilities will be discussed, for instance the second order homogenization of [4]. Here, we shall prefer to apply at the local level a projection technique similar to Lyapunov-Schmidt method. This permits to have a unique solution of the microscopic problem. Various examples

will be discussed, especially beam bending benchmarks; see Fig. 2 and 3. One observes that the appearance of micro-buckling coincides with the loss of ellipticity (point B) and also with the prediction of the shear band criterion.



Fig. 2: (a) Geometry and boundary condition of the macroscopic problem (a bending test); (b) The basic cell with an imperfection of amplitude  $v_0$ .



Fig. 3: (a) The load-displacement diagram; (b) Deformed shapes of microstructure; (c) Macroscopic stress-strain diagram.

- G. Geymonat, S. Müller, N. Triantafyllidis. Homogenization of nonlinearly elastic materials: microscopic bifurcation and macroscopic loss of rank-one convexity. Archive for Rational Mechanics and Analysis, 122, 231-290, 1993.
- [2] S. Nezamabadi, J. Yvonnet, H. Zahrouni, M. Potier-Ferry. A multilevel computational strategy for microscopic and macroscopic instabilities. Computer Methods in Applied Mechanics and Engineering, 198, 2099-2110, 2009.
- [3] B. Budiansky, N. A. Fleck. Compressive failure of fibre composites. Journal of the Mechanics and Physics of Solids, 41, 183-211, 1993.
- [4] V. G. Kousnetzova, M. G. D. Geers, W. A. M. Brekelmans. Multi-scale constitutive modelling of heterogeneous materials with a gradient-enhanced computational homogenization scheme. International Journal for Numerical Methods in Engineering, 54, 1235-1260, 2002.

# Fracture of light alloys with unimodal and bimodal grain size distribution: Multiscale simulation

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Multi-scale computational approach was used for the investigation of damage accumulation and fracture of light titanium, aluminium and magnesium alloys under tension and compression at quasi-static and dynamic loading.

Modified smooth particle hydrodynamics (SPH) method was used for the simulation of grains system on mesoscale level. 3D models of alloys structured representative volumes (RVE) takes into account the influence of unimodal and bimodal grain size distributions, and textures on mechanical properties of alloys.

Fracture of alloys under dynamic loading has probabilistic character and depends on volume concentrations of small and coarse grains. It was found the damage increase at mesoscale level and strength threshold depends on grain size distribution and grain orientations.

Formation of mesocracks can be described as creation a cluster of fractured particles at micro scale level. The increase in ductility of light alloys under quasi-static tension occurs when specific volume of coarse grains is greater than 30 %. The increasing of the macroscopic shear strength and the spall strength of UFG light alloys under growth of strain rates depend not only on the average grain size but a distribution of grain sizes. Fine precipitates in alloys not only affect the hardening but also lead to change the influence of the grains size distribution on volume concentration of shear bands.

#### References

[1] V.A. Skripnyak, E.G. Skripnyak, N.V. Skripnyak, I.K. Vaganova, V.V. Skripnyak Failure mechanisms of light alloys with a bimodal grain size distribution. In: Book 11th. World Congress on Computational Mechanics (WCCM XI), Barselona, July 25-29, 2014, Spain Vol. IV. -P. 3915-3925.

# Double scale approaches for the behaviour of a clay rock: full field measurement and FE<sup>2</sup> model for hydro-mechanical coupling

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In the engineering of rocks, macroscopic behaviour results from mechanisms of deformation at several smaller scales. A complex example is presented by clay rocks, e.g., in the context of nuclear waste storage. Clay rock structures, and thus the mechanisms of deformation, involve different structures across a wide range of scales (from nm to mm), including clay platelets, clay aggregates, mineral inclusions and cracks. These structures and their interactions determine the instantaneous and long term behaviour of the material and its ability to act as a fluid barrier and thus ensure containment of stored waste. As the permeability is influenced by mechanical alteration of the material, the coupling between hydraulic and mechanical behaviour of the host rock is an important factor. This hydro-mechanical coupling on the engineering scale, more especially the permeability change, is controlled essentially by deformation and cracking at the scale of mineral inclusions. In this context, both an experimental fullfield characterization of the deformation at two scales in the clay rock and a doublescale hydromechanical model were developed. In this last, a macro scale is defined at engineering level and a micro scale at the material level [1]. The micro scale computations on a representative elementary volume (REV) provide the local material behaviour for the macro scale. Finite element methods are used to solve both the microscale and the macroscale problem using the so-called finite element squared ( $FE^2$ ) method by computational homogenization.

For the experimental characterization, mechanical tests were performed *in situ* in the nano-tomography beamline ID19 of ESRF (Grenoble). A compression in triaxial conditions at three levels of confining pressure (from 5 to 20 MPa) was applied on small specimens (1.3 mm diameter and 2.5 mm height), with a voxel size of 0.7  $\mu$ m. Digital image correlation (DIC) in volume was used to measure the increment strain tensor fields between different loading steps of tests. The overlapping of the DIC strain fields on the microstructure allows to identify the mechanisms of deformation at the scale of mineral inclusions (a few ten micros size).

Concerning the double-scale model, on the macro level, a poromechanical continuum under quasi-static assumption is defined. To obtain objective (mesh independent) results for strain localization problems, an enriched macroscale continuum

is needed that introduces an internal length in the constitutive law. A local second gradient poromechanical continuum is used [2]. It allows to decompose the constitutive relation in a classical one (stress-strain relation), and a second gradient part that links the second gradient of displacement to its dual term, a double stress. The double-scale model is associated to the classical part, while a phenomenological elastic relation is chosen for the second gradient part of the constitutive relation and implicitly controls the internal length of the model.

On the micro scale, a model for hydromechanical coupling is used for REV with periodic condition, in which elastic solids are separated by cohesive interfaces. The interfaces form a pore channel network that is fully saturated with a compressible fluid. Fluid transport (assumed laminar) is controlled by the conductivity of the channels that depends on the interface openings, and the fluid pressure distribution. A linear-elastic constitutive law is used for the solids, degradation of the material is concentrated in the interfaces. Cohesive forces between the solid grains act normally and tangentially to the interface orientation as a function of the history of respectively the normal and tangential relative displacement of the opposite interface boundaries. Fluid pressure within the interfaces leads to fluid normal forces acting on the solid grains. Boundaries conditions of the periodic REV follow from the macro state variables (displacement gradient, pore pressure and pressure gradient). Reversely, considering a balanced REV, the macro responses (stress and fluid flux and mass) are founded by homogenization as averaged values. The consistent tangent stiffness matrix which is used to solve the linearized macro problem is obtained by homogenization. The computational homogenization by static condensation developed for purely mechanical problem [3] has been extended to hydro-mechanical problems [4, 5].

While such numerically homogenised models allow introducing complexity (anisotropy, damage, history of the loading, hydro-mechanical coupling, non-linear rate response), the challenge will be their identification. The experimental characterisation of the mechanisms of deformation will be a fruitful way to go further.

- [1] Frey, J., Dascalu, C., Chambon, R. A two-scale poromechanical model for cohesive rocks. Acta Geotechnica, 8:2, 107-124, 2013.
- [2] Collin, F., Chambon, R., Charlier, R. A finite element method for poro mechanical modelling of geotechnical problems using local second gradient models. Int. J. Numer. Meth. Engng., 65:11, 1749-1772, 2006.
- [3] Kouznetsova, V., Brekelmans, W., Baaijens, F. An approach to micro-macro modeling of heterogeneous materials. Computational Mech., 27, 37-48, 2001.
- [4] van den Eijnden, B., Collin, F., Bésuelle, P., Chambon, R. A double scale modelling approach for hydromechanical coupling. Proc. 3rd Int. Symp. on Computational Geomechanics, August 21-23, 2013, Krakaw, Poland.
- [5] van den Eijnden, B., Bésuelle, P., Collin, F., Chambon, R. Doublescale modelling of hydromechanical coupling and anisotropy in localization problems, Int. Symp. Geomechanics from Micro to Macro, Sept. 1-3 2014, Cambridge, UK

# A Multiscale Formulation for Cohesive Fracture Analysis

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The contribution presents a multiscale formulation addressed to evaluate cohesive model developed for fracture problems. The multiscale methodology can be categorized as a semi-concurrent model, which uses a Representative Volume Element (RVE) to determine the cohesive forces at the macroscale.

A variational multiscale formulation of the methodology has been previously presented by the authors ([1]). It has been shown there that the approach provides objective macroscale cohesive forces with respect to the microcell size, even after the onset of the strain localization process observed at the macroscopic scale.

Subsequently, the formulation has been generalized and improved in three aspects: 1) cohesive surfaces are introduced at both scales of analysis; they are modeled with strong discontinuity kinematics. New equations describing the insertion of the macro-scale strains, into the micro-scale and the posterior homogenization procedure are considered ([2]); 2) extention of the formulation to model elastoplastic large deformation problems. 3) the computational procedure and numerical implementation have been adapted to the improved formulation; The first and second points are emphasized here, while the third one is briefly summarized in this contribution.

The methodology is numerically assessed through a number of simulations. Typically, quasi-brittle concrete fracture problem are modeled. The numerical simulation adequately captures the material degradation phenomenon at the mesostructural level, which induces the activation of cohesive surfaces at the structural scale.

- P.J. Sanchez, P.J. Blanco, A.E. Huespe, and R.A. Feijóo. Failure-oriented multi-scale variational formulation: micro-structures with nucleation and evolution of softening bands. Comput. Meth. App. Mech. Eng., 257:221–247, 2013.
- [2] S. Toro, P.J. Sanchez, P.J. Blanco, E.A. de Souza Neto, A.E. Huespe, and R.A. Feijóo. Multiscale formulation for material failure accounting for cohesive cracks at the macro and micro scales. submitted.

# Multiscale Homogenization for Ductile Materials with Initial Voids: an Arc Length Based Procedure for the RVE Constitutive Modeling

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When applying multiscale computational homogenization for the constitutive modeling of ductile materials with microscopic voids, the macroscopic stress triaxiality ratio and Lode angle are crucial parameters to be controlled. The basic aim of the homogenization procedure is to model the macroscopic response by accounting for the main features of the material behavior at the microstructural level. The bridge of scales is conducted by firstly constructing the equivalent constitutive response of the so called Representative Volume Element (RVE) of the microstructure. The RVE constitutive response is constructed by incrementally imposing a displacement field to the RVE faces such that prescribed values of the macroscopic stress parameters are kept under control. However, in order to preserve the constitutive response in correspondence to each set of prescribed values for triaxiality ratio and Lode parameters the evaluation of the imposed boundary displacement field requires a specific nonlinear procedure. In this work we propose a numerical nonlinear procedure to construct the RVE constitutive response by exploring both arc length technique and tangent stiffness concepts. The resultant algorithm is based on a prediction-correction strategy. Accordingly, at each step, the initial prescribed displacements applied to each RVE faces are iteratively corrected in order to verify the prescribed values of the macroscopic stress parameters. To simplify the numerical analyses, a cubic RVE with a spherical void at its center is hereby considered. The constitutive response of the material around the void is assumed as elastoplastic von Mises material with isotropic hardening. The equivalent RVE constitutive responses are constructed for different combinations of triaxiality ratio and
Lode angle, so that their influence can be noticed. In addition, the evolution of the plastic strain distribution is represented to put in evidence the onset of strain localization. The numerical results presented shown the effectiveness of the strategy proposed.

# Peridynamic modelling of nanoindentation of thin copper films

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The traditional multiscale approach couples two models operating at different scales. An alternative modelling strategy, called peridynamics originally developed by [1], is to continualize the molecular dynamic models, thus replacing inhomogeneities present on smaller length scales by an enhanced continuum description on larger length scales resulting in a nonlocal reformulation of continuum mechanics. Peridynamics is a single multiscale model valid over wide range of length scales and can be considered as an upscaling of molecular dynamics. Therefore peridynamics models should recover the same dynamics and preserve all characteristic properties of molecular dynamics, which are lost by classical continuum mechanics models. The advantage of the peridynamic models is that they can be solved more cheaply than the corresponding molecular dynamic models. Peridynamics is a generalized continuum theory employing a nonlocal model of force interaction. Each material point interacts with its neighborhood within a sphere, called the horizon that serves as an internal length scale in the model. The interaction between the material points is described by a bond force which is not an electrostatic force but can be related to the strain energy of classical continuum mechanics.

Nanoindentation is useful experimental method to characterize the micromechanical properties of materials and have been used to determine elastic and plastic properties, such Young's modulus and hardness of the material from force-displacement curves. At the nanoscale molecular dynamic models are often used. For many but the smallest systems, molecular dynamic models are computationally too expensive, whereas classical continuum mechanics models not accurately can resolve microscale phenomena. The objective of this study is to employ peridynamics theory and to simulate nanoindentation using LAMMPS [2], with a spherical indenter targeting a copper film. The aim is to show how this approach can be used at the nanoscale. At the nanoscale, the copper coating is modeled as a thin rectangular plate, with the bottom particle layers locked and periodic boundary conditions are applied. Elastic and plastic behaviors are investigated. The force-indentation curves at different load levels have been studied in [3]. From such curves the Young's modulus and the hardening of the material are typically extracted. The deformation is studied for different indentation depths showing a realistic distribution of the particles in agreement with result obtain from MD, including formation of slip patterns below the indenter and typical pile-up effects.

- [1] S. A. Silling, Reformulation of Elasticity Theory for Discontinuities and Long Range Forces. Journal of Mechanics and Physics of Solids, *48*, 175–209, 2000.
- [2] M. L. Parks, P. Seleson, S. J. Plimpton, S. A. Silling, and R. B. Lehoucq, Peridynamics with LAMMPS: A User Guide v0.3 Beta, Sandia Report.
- [3] H. Roca, S. Bourreau. A. Ahadi and S. Melin, Indentation of thin copper film using peridynamics, Proceedings to the 27 th Nordic Seminars on Computational mechanics, NSCM 27, Stockholm, Sweden. 2014.

## A Multiscale Approach to Fracture in Brick-Mortar Masonry Composites

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This study focuses on the fracture behavior in heterogeneous masonry composed of clay brick units that are bonded by mortar joints. There are three main levels of observation to deal with the layered material system in which adhesion of brick-mortar interfaces plays a critical role in fracture initiation and inter/intra fracture propagation in the brick and mortar material system. At the micro/meso level of observation the disparate fracture processes are governed by the mismatch of the constituent properties which introduce curious interactions such as tension in the stiffer brick units under far-field compression which is a remarkable feature of the so-called prism test. In contrast, the so-called triplet test of double lap shear leads to delamination of the brick and mortar interfaces and finally to fracture of the stronger brick substrate. The so-called wrench test is used to evaluate the tensile capacity of adhesive bond that governs separation of bricks from their adherent mortar bed joints. Thereby tension experiments indicate that adhesive debonding is not necessarily a surface dominated process when porous clay bricks are bonded to cement-based mortars.

The paper includes experimental observations of the different fracture processes which are interpreted at the micro-, meso- and macro-scale levels of observation in form of (i) fracture-informed lattice models, (ii) fracture energy-based interface models of zero- vs. finite thickness, and (iii) anisotropic continuum models that exhibit directional fracture energy softening features. The response predictions of the three levels of computational models will be compared and assessed with regard to their predictive limitations and shortcomings to deal with full-scale masonry walls and masonry structures. Thereby, the arguments of fracture initiation and fracture propagation will be the central issue how to separate tensile debonding from shear slip, and how to upscale the underlying fracture properties from lattice-based energy release rates and fracture surface observations to the composite brick and mortar material system. - chapter 11 -



MS14: Nonlocal damage models and other regularized approaches & •Transition from damage to fracture



# Modelling of microcracking in microtomography image-based models of cement materials using phase field method

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In this work [1], crack nucleation and propagation in highly heterogeneous materials models, such as those obtained by micro-CT imagery of real materials, is investigated for the first time by means of the phase field method. The Phase field method [2] is based on a variational formulation of brittle fracture with a regularized approximation of discontinuities [3]. It does not require an explicit tracking of the cracks and avoids the classical mesh sensitivity issues related to smeared cracks models. It also allows crack nucleation. Its various advantages for voxel-based models of microstructures are discussed. More specifically, we show that the resolution related to the initial image and thus to meshes for discretizing the same microstructure does not significantly affect the simulated crack path and provides a quickly convergent response curve. A shifted strain split operator algorithm is proposed to handle unilateral contact within cracks in a very efficient manner. Several 2D and 3D crack nucleation and propagation analyses in highly heterogeneous materials are carried out, with application to microtomography-based segmented images of cementitious materials.

- [1] T.T. Nguyen, J. Yvonnet, M. Bornert, C. Chateau, Q.Z. Zhu. A phase field method to simulate crack nucleation and propagation in strongly heterogeneous materials from direct imaging of their microstructure. Submitted.
- [2] C. Miehe, M. Hofacker, F. Weschinger. A phase field model for rate independent cracks propagation: robust algorithmic implementation base on operator plits. Comput. Meth. Appl. Mech. Eng, 199:2776-2778, 2010.
- [3] G. Francfort, J.J. Marigo. Revisiting brittle fracture as an energy minimizatin problem. J. Mech. Phys. Solids, 46(8):1319-1342, 1998.



Figure 1 : Traction test of a 3D microstructure defined from microtomography image, microcracking propagation for lower end displacement (a) : U = 0 mm, (b) :  $U = 11.10^{-3}$  mm, (c)  $U = 11.2.10^{-3}$  mm, (d)  $U = 11.3 \ 10^{-3}$  mm.



Figure 2 : tension test of a 3D microstructure defined from microtomography image: stress-deflection curve.

# Phase-field modeling of dynamic instabilities in brittle fracture

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Detailed experimental studies of brittle fracture during the past two decades have revealed the existence of dynamic instabilities of fast moving cracks, i.e. cracks propagating at velocities comparable to the Rayleigh wave speed. Depending on the material type and geometry, those instabilities can be manifested in the form of the widely observed microbranching phenomenon or oscillatory crack paths. A fundamental understanding of those instabilities is still missing. Firstly, it is not yet established if those instabilities can be fully described within the standard linear elastic fracture mechanics (LEFM) framework that models dynamic cracks with a velocity-dependent fracture energy and a vanishing mode II stress intensity factor, assuming that the principle of local symmetry remains applicable for fast moving cracks. Secondly, it is still not clear what roles the microscopic details of failure mechanisms within the process zone, sample dimensionality (thin versus thick samples), elastic nonlinearities, and material heterogeneities play in those instabilities.

The phase-field method has emerged as a powerful technique to simulate crack propagation in different geometries and loading conditions [1]. This method is rooted in continuum models of phase transformations, which exploit a coarse-grained scalar order parameter  $\phi$  to distinguish between different phases. In a fracture context,  $\phi$  has been re-interpreted as a phenomenological measure of damage [2], which varies smoothly in space between two values corresponding to the intact and broken states of the material. Furthermore, coupled dynamical equations for the phase and displacement fields can be derived variationally from an energy functional with both elastic strain and surface energy contributions. This approach incorporates both the short scale physics of materials failure and macroscopic elasticity within a set of self-consistent of equations. In addition, those equations scan be simulated on massively parallel computer architecture to describe geometrically complex dynamical phenomena such as crack nucleation, crack kinking [3] and branching, and crack-front segmentation in three dimensions [4].

This talk will describe recent progress to model dynamic instabilities using the phasefield modeling approach. A new phase-field formulation that makes it possible to simulate fast moving cracks up to velocities approaching the Rayleigh wave speed will be presented together with numerical simulations that test the quantitative validity of this model by direct comparisons with LEFM predictions for accelerating cracks [5], see Fig. 1. In addition, results that shed new light on the origin of oscillatory instabilities will be presented [6].



FIGURE 1 – Accelerating tensile cracks in a thin sample of height 2W and width 4W with imposed displacement  $\pm \Delta$  at the vertical boundaries. Left panel : Dimensionless velocity  $v/c_s$  for increasing crack length *l* at three different loadings.  $c_s$  is the shear wave speed. Right panel : Fracture energy  $\Gamma$  as a function of dimensionless velocity. The steady state  $\Gamma$  is measured in a strip with a large width to height aspect ratio when acceleration effects are negligible.

## Références

- R. Spatscheck, E. Brener, and A. Karma, *Phase-Field Modeling of Crack Propagation*, Phil. Mag. **91**, 75–95 (2011).
- [2] A. Karma, H. Levine, and D. Kessler, *Phase-Field Model of Mode-III Dynamic Fracture*, Phys. Rev. Lett. 87, 045501 (2001).
- [3] V. Hakim and A. Karma, *Laws of crack motion and phase-field models of fracture*, J. Mech. Phys. Solids 57, 342–368 (2009).
- [4] A. Pons and A. Karma, *Helical crack-front instability in mixed mode fracture*, Nature 464, 85–89 (2010).
- [5] T. Goldman, A. Livne, and J. Fineberg, *Acquisition of Inertia by a Moving Crack*, Phys. Rev. Lett. **104**, 114301 (2010).
- [6] E. Bouchbinder and I. Procaccia, Oscillatory Instability in Two-Dimensional Dynamic Fracture, Phys. Rev. Lett. 98, 124302 (2007).

## A transient gradient damage model based on the homogenization of inter-granular failure

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It is well documented that standard damage models become ill-posed during strain softening, leading to numerical results which are mesh dependent. One remedy is to incorporate higher-order enhancements to regularize the softening behavior, with the associated length scale parameter collectively characterizing the various microprocesses underlying the localized deformation. However, a clear physical interpretation and decomposition of the length scale parameter is often lacking.

In many higher-order damage models, a constant length scale parameter is adopted, which may result in spurious damage growth at high deformation levels – instead of localizing into a single crack, the damage region expands with deformation. In view of this anomaly, a strain-based transient gradient damage model was proposed in [1], where the nonlocal interaction is controlled by a gradient activity parameter through a phenomenological evolution law.

In this contribution, we focus on the failure of polycrystalline materials which are dominated by inter-granular cracks. Numerically, the fracture process can be modelled by inserting cohesive elements at the grain boundaries. For a large problem, however, such a detailed resolution can be computationally expensive. Motivated by the need for a more efficient model that adequately describes the failure process, we adopt the homogenization theory proposed in [2] and consistently translates the fine-scale intergranular fracture model onto the macro level.

For simplicity, a uniaxial problem is considered. Through the scale transition, a Helmholtz-type homogenized microforce balance is extracted. In contrast to the many gradient damage models in literature which are formulated top-down, our homogenized damage model is obtained bottom-up, hence more predictive, since the fine-scale properties (e.g. grain size, surface modulus) propagate and naturally manifest themselves in the length scale parameter at the macro scale. In addition, the non-local interaction in the microforce balance has a transient nature. A 1D spectral analysis reveals that the softening response is well-regularized, with a vanishing localized bandwidth at complete failure.

### References

[1] M.G.D. Geers, R. de Borst, W.A.M. Brekelmans, R.H.J. Peerlings. Strain-based

transient-gradient damage model for failure analyses. Computer Methods in applied mechanics and engineering, 160, 133-153, 1998.

[2] L.H. Poh. Scale transition of a higher order plasticity model – A consistent homogenization theory from meso to macro. Journal of the Mechanics and Physics of Solids, 61, 2692-2710, 2013.

# A Geometry-Based Anisotropic Gradient-Enhanced Damage Model for Modelling Masonry Failure

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While masonry is known for its simple method of construction, its computational analysis poses many challenges. Since masonry consists of two distinct materials, i.e. mortar joints and clay bricks, the use of a suitable and efficient numerical model is essential in order to accurately model the orthotropic structural behaviour. Moreover, due to the quasi-brittle nature of both constituents, a numerical masonry model should incorporate a robust and objective description of strain localisation and material failure.

In our contribution, we propose a novel modelling approach in which failure of the orthotropic masonry composite is modelled in a continuous manner. Unlike traditional continuous masonry models, in which mesh-dependence of strain softening is regularised using Cosserat continua [1] or non-local models [2], an implicit gradient-enhanced damage model is used for the objective description of strain localisation and failure.

In contrast to conventional isotropic gradient-enhanced damage models [3], an anisotropic interaction kernel is employed, which size and shape is not only governed by the equivalent stress level [4], but also by the underlying geometry of the structure, i.e. the masonry bond. Through several numerical examples, the proposed masonry model will be compared with detailed mesoscale models in which joints are explicitly modelled [5]. Special attention will also be addressed to the influence of the model parameters and their interaction with the orthotropic elastic properties.

- D. Addessi, E. Sacco, and A. Paolone. Cosserat model for periodic masonry deduced by nonlinear homogenization. *European Journal of Mechanics A/Solids*, 29, 724-737, 2010.
- [2] S. Marfia and E. Sacco. Multiscale damage contact-friction model for periodic masonry walls. *Comput. Methods Appl. Mech. Engrg.*, 205-208, 189-203, 2012.

- [3] R. H. J. Peerlings, R. de Borst, W. A. M. Brekelmans, and J. H. P. de Vree. Gradientenhanced damage for quasi-brittle materials. *Int. J. Numer. Meth. Engrg.*, 39, 3391-3403, 1996.
- [4] A. Simone. Advances in gradient-enhanced damage models with evolving length scales. In *Proceedings of The Third International Conference on Computational Modeling of Fracture and Failure of Materials and Structures*, Prague, 2013.
- [5] B. Vandoren, K. De Proft, A. Simone, and L. J. Sluys. Mesoscopic modelling of masonry using weak and strong discontinuities. *Comput. Methods Appl. Mech. Engrg.*, 255, 167-182, 2013.

# Numerical treatment of the damage/crack transition in a Gurson ductile material

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The present work aims at the numerical treatment of the crack propagation in engineering materials whose failure results from void initiation, growth and coalescence. With this aim in view, the objective is herein to describe the intermediate stage between more or less diffuse damage and the initiation of a macroscopic crack. This transient stage of damage induced deformation localisation in a narrow band necessitates a specific numerical treatment.

Previous works (see Longère et al. [1]) have evidenced the interest of modifying the Gurson-Tvergaard-Needleman (GTN), (ductile) damage-(visco)plasticity, constitutive model for reproducing the void growth induced damage in shear, in addition to the combined effects of strain hardening, thermal softening and viscoplasticity.

Recent works have shown the feasibility of a coupling between the aforementioned damage-plasticity model and the eXtended Finite Element Method (X-FEM), both implemented as user subroutines (UMAT and UEL, respectively) into the engineering finite element computation code Abaqus (see Crété et al. [2]).

In order to reduce the inherent mesh dependency in the pre-failure, localization process induced softening regime, the finite element formulation is enriched by embedding a finite thickness band (embedded finite element method, E-FEM) into the element (see e.g. Huespe et al. [3]). The orientation of the localization band is accordingly determined by a bifurcation analysis (see Rice [4]).

The passage from the localized band of finite width to the crack of (initially) null width is characterized by an empirical transition law relating to experimental results. At the time of crack initiation, the element formulation is switched from an E-FE to an X-FE formulation; an efficient numerical procedure is presented.

The performance of the methodology is evaluated considering the numerical simulation of the tension loading of a notched plate.

- P. Longère, A.G. Geffroy, B. Leblé and A. Dragon, 2012, Modelling the transition between dense metal and damaged (micro-porous) metal viscoplasticity. Int. J. Dam. Mech., 21, 1020-1063
- [2] J. P. Crété, P. Longère and J.M. Cadou, 2014, Numerical modelling of crack propagation in ductile materials combining the GTN model and X-FEM. Comp. Methods Appl. Mech. Eng., 275, 204-233
- [3] A.E. Huespe, A. Needleman, J. Oliver and P.J. Sanchez, 2009, A finite thickness band method for ductile fracture analysis. Int. J. Plast., 25, 2349-2365
- [4] J.R. Rice, 1976, The localization of plastic deformation, in W.T. Koiter ed., Theoretical and Applied Mechanics, Koiter, pp.207-220,

## Fracture as material instability in strain-softening processes: a non-convex variational approach

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A variational approach to strain-gradient plasticity is proposed, which describes cohesive fracture as a bulk process of progressive strain localization, concluding with a final rupture. As in [1], the model accounts for an additive decomposition of the deformation into an elastic and a plastic part, and it is based on an energy functional, sum of a stored elastic bulk energy, a non-convex plastic energy, which is supposed to be totally dissipative, and a quadratic non-local gradient term. The second and third terms depend on the cumulated plastic strain. The model [1] is enriched by assuming that the coefficient of the non-local energy depends on the cumulated plastic strain, in order to improve the description of fracture. The evolution of the deformation is described by a two-field incremental minimization problem, where the displacement and the plastic deformation are assumed as independent variables. To provide a thorough mechanistic understanding, the model derivation is done in a one-dimensional mathematical setting. The approach can be readily extended to three-dimensional isotropic softening gradient plasticity, or to a multi slip strain gradient crystal plasticity setting.

In the present communication, attention is focused on the model capability of describing processes of strain localization concluding with fracture, which are induced by non-convex plastic energies. Fracture represents an instable configuration of the incremental minimization problem governing the evolution. In [1] it was found that an equilibrium solution is unstable if at least one eigenvalue of a certain eigenvalue problem associated to the variational problem is negative. In this case of negative eigenvalue, the second variation of the problem functional can attain unlimited negative values for perturbations concentrated on zones of the body of sufficiently small size. This situation characterizes fractured configurations in a variational way. Conditions for the stability loss are analyzed, and analytical expressions involving the problem constitutive parameters are proposed.

The predicted fracture mechanism points out a crucial idea of the proposed model, i.e., the tentative of describing cohesive fracture as a bulk process of progressive strain localization, which concludes with a final material instability. This approach is motivated by the observation that the fracture of ductile materials is characterized by the formation of the so-called *process zone*, that is a region where strains localize and the material becomes weaker and weaker, and only at the very end strains coalesce in

fracture surfaces. Accordingly, the plastic energy density can be also called cohesive energy as in [1], and the proposed approach presents as an alternative to the classical cohesive theories, which consider fracture process as a surface phenomenon.

Some simulations are proposed to illustrate the fracture mechanism above described, and to show the close relation which exists between the shape of the plastic energy and the predicted behaviors. In particular, the evolution of the deformation in metals subject to uniaxial loading is reproduced. The distinct phases of the evolution observed in experiments are found: from the initial yielding phase, in which inelastic deformations form, and propagate as slow plastic waves, to the final rupture, which constitutes the ending stage of a strain-localization process. In Fig. 1(a) the force-elongation curve of a tensile bone-shaped steel bar is reproduced by assuming a convex-concave plastic energy. The deformation corresponding to different points of the evolution is represented in Fig. 1(b).



Fig. 1. (a) Force-elongation curve of a bone-shaped steel bar subject to tensile loading. (b) Deformation field at different points of the evolution.

### References

 G. Del Piero, G. Lancioni, R. March, A diffuse cohesive energy approach to fracture and plasticity: the one-dimensional case", Journal of Mechanics of Material and Structures, 8(2-4), 109-151, 2013

# Mixing first- and second-gradient models in finite element simulations of ductile rupture

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An interesting second-gradient model for plastic porous solids, extending Gurson's classical first-gradient model [1] for porous ductile solids, was proposed some years ago by Gologanu et al. [2] in order to settle the issue of unlimited localization of strain and damage and the ensuing mesh sensitivity in finite element calculations. This model was successfully implemented in the finite element code SYSTUS® by Enakoutsa and Leblond [3] by introducing extra nodal degrees of freedom (DOF) representing the components of the strain tensor. One important drawback of this procedure, however, was an awkwardly large number of DOF, notably in 3D, which generated various difficulties, especially with regard to convergence of the global elastoplastic iterations

A new implementation solving this problem was very recently proposed by Bergheau et al. [4]. The central point of the new algorithm was a procedure of elimination of the nodal DOF representing the strain components, which permitted to reduce the number of DOF per node to its standard value. This was achieved by writing the equality of the new nodal DOF and the strain components in a weak sense; the "mass matrix" appearing in the left-hand side of the vectorial relation thus obtained was then lumped and inverted straightforwardly, so as to relate the new DOF to the nodal displacements explicitly. The reduction of the number of DOF was found to be quite beneficial to the convergence of the global elastoplastic iterations.

However, a mixing of first- and second-gradient models would be desirable in order to reduce computation time and cost, and reasonable at least in simulations of ductile rupture, where second-gradient models are needed only in zones of limited extent, where strain and damage tend to concentrate. Bergheau et al.'s implementation [4] offers an interesting perspective in this context, since the elimination of nodal DOF representing strain components means that exactly the same DOF are used for first- and second-gradient models. This makes the mixing very easy by eliminating the need for development of special transition elements and requiring only elementary, low-level modifications of the code; such modifications essentially reduce to introduction of a

switch at the beginning of the loop over elements, the role of which is to call the appropriate routine to solve the constitutive equations, according to whether the element considered obeys a first- or second-gradient model.

This procedure is illustrated in the present paper by comparing full second-gradient and mixed first/second gradient simulations of a CT specimen made of a ductile material and undergoing full rupture. The calculations are performed in 2D and plane strain but extension to 3D cases would be straightforward. The two types of simulations are checked to yield almost identical results, with lower computation time and cost for the second one.

- A.L. Gurson. Continuum theory of ductile rupture by void nucleation and growth: Part I - Yield criteria and flow rules for porous ductile media. ASME Journal of Engineering Materials and Technology, 99, 2-15, 1977.
- [2] M. Gologanu, J.B. Leblond, G. Perrin, J. Devaux. Recent extensions of Gurson's model for porous ductile metals. In: Continuum Micromechanics, P. Suquet, ed., CISM Courses and Lectures No. 377, Springer, pp. 61-130, 1997.
- [3] K. Enakoutsa, J.B. Leblond. Numerical implementation and assessment of the GLPD micromorphic model of ductile rupture. European Journal of Mechanics A/Solids, 28, 445-460, 2009.
- [4] J.M. Bergheau, J.B. Leblond, G. Perrin. A new numerical implementation of a second-gradient model for plastic porous solids, with an application to the simulation of ductile rupture tests. Computer Methods in Applied Mechanics and Engineering, 268, 105-125, 2014.

# A transition approach from a local second gradient model to a cohesive zone model

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Among the various regularization methods that enable to deal with the problem of strain localization in the framework of continuum mechanics, the class of generalized continua introduces an internal length by taking into account additional kinematics terms into the virtual power principles. As a particular case of this type of continua, the local second gradient model developed by Chambon and coworkers [1] has been successfully used for soils where the main problem is shear banding. More recently, it has been also used in the context of damage mechanics for concrete structural elements [2] where strains mainly localize under the first mode of crack propagation.

We show that this type of strain localization can become problematic for the second gradient model. Indeed, the forces transmitted through the crack do not necessarily vanish for high values of damage. This is due to the the additional stresses, the so-called double stresses, of the second gradient model. Another problem, common among regularization techniques, is the continuous spreading of the localization band again for similarly high damage values (fully formed cracks).

Following the work by Comi et al [3] and Cuvilliez et al [4] for different regularization methods, we propose to deal with theses issues applying a transition to a cohesive zone model. In this way, the various phases of strain localization and fracture can be correctly modeled : from diffuse damage to localization till finally to a fully formed crack. The theoretical formulation and finite element implementation are discussed with special care to the additional boundary conditions which naturally appear in a second gradient continuum. Application to a 3 point bending test on a notched beam illustrates the limitations and the advantages of the approach. The results show that the new model is more adequate to represent the full experimental force displacement curve.



FIGURE 1 - 3 point bending test : Damage distribution at the notch tip with open cohesive zones

## Références

- [1] R. Chambon, D. Caillerie, T. Matsushima. Plastic continuum with microstructure, local second gradient theories for geomaterials : localization studies. *International Journal of Solids and Structures*, 38(46-47), 8503-8527, 2001.
- [2] G. Jouan, P. Kotronis, F. Collin. Using a second gradient model to simulate the behaviour of concrete of concrete structural elements. *Finite Elements in Analysis and Design*, 90(0), 50-60, 2014.
- [3] C. Comi, S. Mariani, U. Perego. An extended fe strategy for transition from continuum damage to mode I cohesive crack propagation. *International Journal for Numerical and Analytical Methods in Geomechanics*, 31(2), 213-238, 2007.
- [4] S. Cuvilliez, F. Feyel, E. Lorentz, S. Michel-Ponnelle. A Finite element approach coupling a continuous gradient damage model and a cohesive zone model within the framework of quasi-brittle failure.*Computer Methods in Applied Mechanics and Engineering*, 237, 244-259, 2012.

# Microdamage modeling of crack initiation and propagation in metal single crystals

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Single crystal components operating at elevated temperatures are subjected to severe thermomechanical loading conditions. A major issue is to develop models to predict crack initiation and crack growth in the presence of strong stress and temperature gradients. The strongly anisotropic elastoviscoplastic behaviour of the material which is a single crystal nickel base superalloy, must be taken into account. The corresponding model should be able to account for anisotropic crack growth and crack bifurcation in complex stress fields. Moreoever the model must be capable of predicting not only the crack growth rate but also the non-straight crack paths. Anisotropic damage mechanics is a well-suited theoretical framework for the development of crack growth models in single crystals. A model coupling crystal plasticity and cyclic damage has been developed, that shows the interest of the approach, but also its current limits, in particular the strong mesh dependence of the results. Recent developments of nonlocal models within the framework of the mechanics of generalized continua could help overcoming these diculties. A large experimental basis exists concerning initiation and crack growth in single crystal nickel base superalloys. Finite element simulations of the thermomechanics of turbine blades provide detailed information about stress and plastic strain distribution, in particular near geometrical singularities like cooling holes and slits. First of all, on the basis of crystal plasticity theory which provides a solid link between stress and plastic strains, an uncoupled damage mechanics model based on the history of FE calculations will be presented. Afterwards, an incremental damage model based on generalized continua will be proposed and model predictions for the initiation and growth of microcracks by solving the mesh dependency, will be discussed.

- [1] S. Forest. The micromorphic approach for gradient elasticity, viscoplasticity and damage. *ASCE Journal of Engineering Mechanics*, 135:117–131, 2009.
- [2] O. Aslan, S. Quilici, and S. Forest. Numerical modeling of fatigue crack growth in single crystals based on microdamage theory. *International Journal of Damage Mechanics*, 20:681–705, 2011.
- [3] O. Aslan, N. M. Cordero, A. Gaubert, and S. Forest. Micromorphic approach to single crystal plasticity and damage. *International Journal of Engineering Science*, 49:1311–1325, 2011.
- [4] J. Li, H. Proudhon, A. Roos, V. Chiaruttini, and S. Forest. Crystal plasticity finite element simulation of crack growth in single crystals. *Computational Materials Science*, 90:191–197, 2014.

# Algorithmic aspects of the efficient phase-field computing of fracture

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The phase-field approach for modeling fracture phenomena in elastic solids is a very promising framework which has gained popularity within the last decade, see the review paper [1] and the references therein.

In practice, already a two-dimensional quasi-static phase-field formulation is computationally quite demanding. Indeed, the need to resolve the small length scale calls for extremely fine meshes, at least locally, in a crack phase-field transition zone. Also, due to non-convexity of the free-energy functional, a robust, but a priori *non-efficient* staggered solution scheme based on algorithmic decoupling is typically used, e.g. in [2, 3, 4].

We discuss some aspects of efficient (i.e. fast, yet accurate) quasi-static phase-field computing of (brittle) fracture, including the use of a monolithic solution scheme along with adaptive (error-controlled) mesh refinement strategy [5].

- M. Ambati, T. Gerasimov, L. De Lorenzis. A review on phase-field models of brittle fracture and a new fast hybrid formulation. *Computational Mechanics*, DOI 10.1007/s00466-014-1109-y, 2014.
- [2] B. Bourdin, G.A. Francfort, J.J. Marigo. The variational approach to fracture. *Journal* of *Elasticity*, 91, 5-148, 2008.
- [3] C. Kuhn, R. Müller. A continuum phase field model for fracture. *Engineering Fracture Mechanics*, 77, 3625-3634, 2010.
- [4] C. Miehe, M. Hofacker, F. Welschinger. A phase field model for rate-independent crack propagation: robust algorithmic implementation based on operator splits. *Computer Methods in Applied Mechanical Engineering*, 199, 2765-2778, 2010.
- [5] T. Gerasimov, L. De Lorenzis. Fast and reliable phase-field modeling of fracture using the monolithic scheme and adaptive mesh refinement, *in preparation*.

## **Phase-field modeling of ductile fracture**

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Phase-field modeling of *brittle* fracture [1-5] is a well-established framework that overcomes the limitations of the classical Griffith theory in the prediction of crack nucleation and in the identification of complicated crack paths including branching and merging. In this work, we propose a novel phase-field model for ductile fracture of elasto-plastic solids in the quasi-static kinematically linear regime [6]. The proposed formulation enables to capture the entire failure mechanism of a ductile material exhibiting J2-plasticity, encompassing plasticization, crack initiation, propagation and failure. Several examples demonstrate the ability of the model to reproduce some important phenomenological features of ductile fracture as reported in the experimental literature. An example is illustrated in Figure 1 (I-shaped specimen in tension), our model predicts the crack initiation in the center of a specimen, followed by the successive branching of the band, the bifurcation (loss of symmetry) and the final specimen failure. Beside the new formulation, several examples are discussed, including some with our own experimental verification.



Figure 1: I-shaped specimen under tension: (a) Equivalent plastic strain field at the onset of cracking and (b-d) crack phase-field at various fracture stages.

#### References

[1] B. Bourdin, G.A. Francfort, J.J. Marigo. The variational approach to fracture. *Journal of Elasticity*, 91, 5-148, 2008.

- [2] C. Kuhn, R. Muller. A continuum phase field model for fracture. *Engineering Fracture Mechanics*, 77, 3625-3634, 2010.
- [3] C. Miehe, M. Hofacker, F. Welschinger. A phase field model for rate-independent crack propagation: robust algorithmic implementation based on operator splits. *Computer Methods in Applied Mechanics and Engineering*, 199, 2765-2778, 2010.
- [4] M.J. Borden, C.V. Verhoosel, M.A. Scott, T.J.R. Hughes, C.M. Landis. A phase-field description of dynamic brittle fracture. *Computer Methods in Applied Mechanics and Engineering*, 217-220, 77-95, 2012.
- [5] M. Ambati, T. Gerasimov, L. De Lorenzis. A review on phase-field models of brittle fracture and a new fast hybrid formulation. *Computational Mechanics*, DOI 10.1007/s00466-014-1109-y, 2014.
- [6] M. Ambati, T. Gerasimov, L. De Lorenzis. Phase-field modeling of ductile fracture. *Submitted*.

# Isogeometric collocation for phase-field modeling of brittle fracture

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Phase-field modeling of brittle fracture [1,2] is a framework that overcomes the limitations of the classical Griffith theory in the identification of complicated crack paths, branching and coalescence of pre-existing cracks and crack nucleation. The sharp limit case is recovered for a small length-scale parameter, controlling the interface width between fully broken and undamaged phases of a material. From a computational standpoint, this calls for extremely fine meshes in order to accurately resolve the crack pattern obtaining the correct structural response [3,4]. Isogeometric Galerkin formulations attaining higherorder convergence [3,5] have already been proposed and successfully demonstrated. On the other hand, isogeometric collocation has recently emerged as a very promising technology to achieve higher-order convergence while avoiding the cost of quadrature [6] and the first attempts to combine it with phase-field modeling of the Cahn-Hilliard equation have shown interesting results [7]. In this work, we apply the isogeometric collocation framework to phase-field modeling of brittle fracture using the 'hybrid model' developed in our previous paper [8] as well as adopting an alternative interpretation of collocation as Galerkin with reduced integration, aiming at a drastic reduction of the computational cost without sacrificing the accuracy. A comparison with Galerkin approximations is performed.

- [1] B. Bourdin, G.A. Francfort and J.J. Marigo. The variational approach to fracture. *Journal of Elasticity*, 91, 5-148, 2008.
- [2] C. Miehe, F. Welschinger and M. Hofacker. Thermodynamically consistent phasefield models of fracture: variational principles and multi-field FE implementations. *International Journal of Numerical Methods in Engineering*, 83, 1273-1311, 2010.

- [3] M.J. Borden, C.V. Verhoosel, M.A. Scott, T.J.R. Hughes and C.M. Landis. A phase-field description of dynamic brittle fracture. *Computer Methods in Applied Mechanics and Engineering*, 217-220, 77-95, 2012.
- [4] C. Miehe, M. Hofacker and F. Welschinger. A phase field model for rate-independent crack propagation: robust algorithmic implementation based on operator splits. *Computer Methods in Applied Mechanics and Engineering*, 199, 2765-2778, 2010.
- [5] M.J. Borden, T.J.R. Hughes, C.M. Landis and C.V. Verhoosel. A higher-order phasefield model for brittle fracture: Formulation and analysis within the isogeometric analysis framework. *Computer Methods in Applied Mechanics and Engineering*, 273, 100-118, 2014.
- [6] F. Auricchio, L. Beirão da Veiga, T.J.R. Hughes, A. Reali and G. Sangalli. Isogeometric Collocation Methods. *Mathematical Models and Methods in Applied Sciences*, 20(11), 2075-2107, 2010.
- [7] H. Gomez, A. Reali and G. Sangalli. Accurate, efficient, and (iso)geometrically flexible collocation methods for phase-field models. *Journal of Computational Physics*, 262, 153-171, 2014.
- [8] M. Ambati, T. Gerasimov, L. De Lorenzis. A review on phase-field models of brittle fracture and a new fast hybrid formulation. *Computational Mechanics*, DOI 10.1007/s00466-014-1109-y, 2014.

# Gradient-enriched models: the role of boundary conditions on failure initiation and propagation

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Gradient-enriched models are a well established approach to regularise softening failure, but they exhibit some undesired features. One of them is incorrect damage initiation and propagation, see [1]. For a tensile test on a cracked specimen, for instance, a standard gradient model does not predict damage initiation at the crack tip (as suggested by intuition and observed in experiments), but away from it. For a shear test, on the other hand, a non-physical shift of the shear band is predicted.

We show in this work that these unrealistic responses are caused by the choice of boundary conditions for the gradient enrichment. The regularisation equation linking the local equivalent strain e with the non-local counterpart  $\tilde{e}$  is typically complemented with homogenous Neumann boundary conditions:

$$\widetilde{e}(\boldsymbol{x}) - \ell^2 \nabla^2 \widetilde{e}(\boldsymbol{x}) = e(\boldsymbol{x}) \text{ in } \Omega$$
(1)

$$\nabla \widetilde{e} \cdot \boldsymbol{n} = 0 \text{ on } \partial \Omega \tag{2}$$

Regarding damage initiation, and for illustrative purposes, consider a planar crack in an infinite plate under mode I, see Figure 1(a). Although the local field e reaches its peak value at the crack tip (where strains are, in fact, singular), see Figure 1(b), the non-local field  $\tilde{e}$  is maximum away from the crack tip, see Figure 1(c), and damage initiation is wrongly predicted.



Figure 1: Crack under mode I. (a) Problem statement; Mises equivalent strain fields: (b) local; (c) non-local with standard model; (d) non-local with smoothed displacements

One possible remedy for this undesired response would be to choose different boundary conditions. If one replaces equation (2) with Dirichlet boundary conditions (i.e.  $\tilde{e}(\mathbf{x}) = e(\mathbf{x})$  on  $\partial\Omega$ ), then the maximum value of  $\tilde{e}$  remains at the crack tip. With these new boundary conditions, however, other required features of the gradient enrichment, such as smoothing along the boundary, is lost.

In view of this, a different solution is proposed here: to enrich the displacement field, rather than the equivalent strain field. Working with a vectorial, rather than a scalar, field means that there is more flexibility in the choice of boundary conditions. A number of options were tested and compared in [2], and it was concluded that the best choice are so-called combined boundary conditions (i.e. Dirichlet for the normal component and Neumann for the tangential component, in both cases non-homogeneous), so the gradient enrichment is

$$\widetilde{\boldsymbol{u}}(\boldsymbol{x}) - \ell^2 \nabla^2 \widetilde{\boldsymbol{u}}(\boldsymbol{x}) = \boldsymbol{u}(\boldsymbol{x}) \text{ in } \Omega$$
$$\widetilde{\boldsymbol{u}} \cdot \boldsymbol{n} = \boldsymbol{u} \cdot \boldsymbol{n} \text{ on } \partial \Omega$$
$$\nabla(\widetilde{\boldsymbol{u}} \cdot \boldsymbol{t}) \cdot \boldsymbol{n} = \nabla(\boldsymbol{u} \cdot \boldsymbol{t}) \cdot \boldsymbol{n} \text{ on } \partial \Omega$$

Figure 1(d) shows the non-local equivalent strain field  $\tilde{e}$  computed from the non-local displacement field  $\tilde{u}$ , which attains it peak value at the crack tip. At the same time, this gradient enrichment also satisfies the four following criteria, see [2]: reproducibility of order 1 (i.e. a linear local displacement field leads to a linear non-local displacement field); smoothing along the boundary; local response normal to boundaries; volume preservation.

The performance of the proposed approach regarding damage propagation will also be discussed.

- A. Simone, H. Askes and L.J. Sluys. Incorrect initiation and propagation of failure in non-local and gradient-enhanced media. *International Journal of Solids and Structures*, 41(2), 351-362, 2004.
- [2] E. Tamayo-Mas and A. Rodríguez-Ferran. A medial-axis-based model for propagating cracks in a regularised bulk. *International Journal for Numerical Methods in Engineering*, 101(7), 489-520, 2014.

# Strain-gradient vs. damage-gradient regularizations of damage models

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Damage models describe material failure by introducing an additional internal variable modulating the elastic stiffness and inducing an internal energy dissipation in the material. It is well accepted that in order to model the material failures observed in quasibrittle materials, damage models must include stress-softening, *i.e.* the reduction of the admissible stress domain for increasing damage. It can be shown that this constitutive property is associated to the lost of the uniqueness of the solution and the appearance of localized states, a key aspect of the behavior of real structures. It is also widely recognized that in order to preserve the mathematical consistency and the capability of formulating mesh-independent numerical approximation, strain-softening damage models must be regularized by introducing some sort of non-local effects. Among the several regularization strategies proposed in the literature, we may first distinguish between those based on the introduction of smoothed variables in the form of convolution integrals and those penalizing extreme localizations through gradient terms. In [1], after a thorough analysis of the possible nonlocal regularization strategies, is it concluded that only a small subset of the methods proposed in the literature are really effective in providing (thermodynamically) well-posed problems and the use of gradient approaches over the use of convolution integrals is suggested.

One can further classify the possible approaches between those introducing the regularization through Damage Gradient terms (DG) and those using gradient of the elastic Strain (SG), see for instance [2]. We limit our analysis to this last classification and compare how DG and SG damage models compare to the limit Griffith model of brittle fracture.

In particular, the two following forms of the total energy are considered:

$$\mathcal{E}_{DG}(\varepsilon, \alpha) = \int_{\Omega} \left( \frac{1}{2} \operatorname{E}(\alpha) \varepsilon^2 + \operatorname{w}(\alpha) + \operatorname{E}_0 \ell^2 (\alpha')^2 \right) dx, \tag{1}$$

relative to a Damage-Gradient regularized damage model, see for instance [3], and

$$\mathcal{E}_{SG}(\varepsilon, \alpha) = \int_{\Omega} \left( \frac{1}{2} \operatorname{E}(\alpha) \varepsilon^2 + \operatorname{w}(\alpha) + \operatorname{E}_0 \eta^2 (\varepsilon')^2 \right) dx, \tag{2}$$

relative to a Strain-Gradient regularized damage model. Here  $\varepsilon$  and  $\alpha$  mean respectively the strain and the damage fields, whilst  $w(\alpha)$  measures the dissipation due to damage. An undamaged material corresponds to  $\alpha = 0$  and  $E(0) = E_0$ , a completely damaged material to  $\alpha = 1$  and E(1) = 0.

In the first case, (1), the square norm of the damage gradient  $\alpha'$  is penalized through the characteristic length  $\ell$ ; this is known to be sufficient in order to inhibit uncontrolled damage localizations. In the latter case, (2), the gradient of damage is not explicitly penalized and, indeed, the damage field is even allowed to be discontinuous; however a damage discontinuity would imply a strain gradient that is penalized through the characteristic length  $\eta$ . Hence we show that also the SG regularizing term in (2) is able to inhibit uncontrolled damage localizations.



Figure 1: Evolutions of damage for a SG model in 1D case.

However, we also show that while the DG model (1) can be used to as a phase-field regularization of brittle fracture [3], the same is not possible with SG models (2). Indeed in DG models, the completely damaged zones remain localized in bands of finite width not increasing with loading, and the associated energy dissipation can be identified with the material toughness. In SG models, the width of fully damaged zones increase with the loading and it is not possible to associate them to a finite and constant energy dissipation, similar to that of Griffith models of brittle fracture. We conclude that SG models (2), despite being well-posed for their regularizing properties, cannot be consider energetically equivalent to brittle fracture and cannot be regarded as phase-field approximations of Griffith models.

- [1] E. Lorentz and S. Andrieux, Analysis of non-local models through energetic formulations. *Int. J. Solids and Structures* 40:2905–2936, 2003.
- [2] R.H.J. Peerlings, R. De Borst, W.A.M. Brekelmans and J.H.P. De Vree, Gradient Enhanced damage for quasi-brittle materials, *Int. J. Num. Meth. Eng.*, 39:3391–3403, 1996.
- [3] K. Pham, H. Amor, J.J. Marigo and C. Maurini, Gradient damage models and their use to approximate brittle fracture, *Int. J. Damage Mechanics* 20(4):618–652, 2011.

# Intermittency and localization in quasi-brittle failure of heterogeneous materials: A numerical and analytical study

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Quasi-brittle failure of heterogeneous materials can be seen as a complex three steps process. At sufficiently small loading, the material first display an elastic response. Above a certain loading threshold, progressive microcracking occurs, leading to a deviation from the linear behavior. During this phase, strong intermittency and spatial organization of damage are observed both numerically [1] and experimentally [2]. Finally, catastrophic failure occurs associated with spatial localization of damage. Hence, two main challenges in the study of failure by microcracking are (i) the prediction of the critical loading at which this catastrophic failure occurs (ii) the quantitative understanding of the origin of the complex temporal fluctuations and spatial organization of damage.

Our objective is to build a continuum model of quasi-brittle fracture containing a minimum set of ingredients that is able to capture the main features. A first and original ingredient in the proposed model is material heterogeneities that are introduced through an heterogeneous fracture energy field drawn from a uniform distribution. The other key ingredient is the elastic interaction at a structural level between damaged elements that is controlled through a nonlocal approach [3] using a prescribed interaction kernel. This interaction kernel describes how the stress (and so the driving force for damage) is redistributed in the material after some part of the material gets damaged. Finally, damage is assumed to affect only elastic properties: locally, the Young modulus decreases as damage increases. Considering an energy criterion for damage growth, a nonlocal evolution equation is obtained for the damage field in the material under displacement controlled loading conditions. Since we use a thermodynamically consistent approach, our model is able to describe how mechanical energy is transferred into damage energy through bursts of failure events until catastrophic failure occurs. In particular, since material heterogeneity results in a damage structure that is intermittent in time and heterogeneous in space, we will focus on the energy of these localized failure events and their spatial extension.

The proposed approach is first applied to a 1D problem where we *impose* the stress redistribution after damage events. In that case, the interaction kernel is chosen to decay exponentially fast with distance and the interaction length is varied in order to study its effect on the failure behavior of the material. The evolution equation of the damage field is solved numerically, and is used to predict the strain-stress response of the material as it progressively damages. This simple formulation allows to retrieve qualitatively the precursors main features as observed in experiments and finite element simulations [1, 2], independently of the value of the internal length: first, damage evolution is characterized by sudden failure events spreading a large range of energy, which follow a power law distribution. Morevover, these failure events are spatially localized and correlate over some length scale that rapidly increases as complete failure is approached. Finally, complete failure of the material is achieved through a catastrophic damage event that spreads over the whole system. To rationalize these results, we study analytically the onset of catastrophic failure by performing a linear stability analysis of the damage evolution: the damage field is slightly perturbed by some sinusoidal mode around its mean value that corresponds to the equilibrium condition at some prescribed value of the displacement. Catastrophic failure then takes place when the perturbation mode grows in an unstable manner. This analysis gives the critical loading at failure as well as the wavelength of the most unstable mode that are shown to be directly related to the shape of the stress redistribution kernel. Comparison of these analytical predictions with numerical results for systems with low disorder levels shows excellent agreement. It also reveals the key role of the redistribution kernel: by changing its shape, localization can take place before peak load of the stress-strain response is reached, or later in the post-peak regime.

In a second part, the same approach is applied to an elasto-damageable solid under pure shear or uniaxial tension with plane-strain conditions. The local material elastic constants are related with the damage level from micromechanics calculations [4]. In that case, the redistribution kernel can be calculated *rigorously*, and used to predict the onset of instability as well as the angle of the localization bands. These predictions are discussed and compared with direct numerical simulations of localization in progressively damaged solids.

- [1] L. Girard, D. Amitrano and J. Weiss. Failure as critical phenomenon in a progressive damage model. *Journal of Statistical Mechanics*, P01013, 2010.
- [2] A. Guarino, A Guacimartín and S. Ciliberto. An experimental test of the critical behavior of fracture precursors, *European Physical Journal B*, 6, 13-24, 1998.
- [3] Z.P. Bažant and M. Jirásek. Noncal integral formulations of plasticity and damage: a survey of progress, *Journal of Engineering Mechanics*, 128(11), 1119-1149, 2002.
- [4] P. Ponte Castaneda and J. R. Willis. The effect of spatial distribution on the effective behavior of cracked media, *Journal of the Mechanics and Physics of Solids*, 43:1919, 1995.

# Formulation of a constitutive law with a gradual transition from continuous to discontinuous descriptions of cracks in concrete

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Fracture is the salient feature of concrete-like materials. It is responsible for the both strength and stiffness reduction and precedes the structure failure. At the beginning, a thin zone composed of diffused micro-cracks is formed, called a fracture process zone (FPZ) or a localized zone. Later, during further deformation, a distinct discrete macro-crack emerges. An adequate description of cracks in FE calculations is extremely important to obtain physically realistic results.

Within continuum mechanics, there exist two main approaches to describe cracks. The first one describes them in a smeared sense as localized zones of micro-cracks with a certain finite width. The material behaviour may be described using e.g. elasto-plastic, damage mechanics or smeared crack and coupled constitutive laws enhanced with a characteristic length of the microstructure. As an alternative, displacement jumps (discontinuities) along cracks may be introduced while keeping the remaining region as a continuous one. The oldest solutions used interface elements which were defined along finite element edges. The modern ones allow for considering displacement jumps in the interior of finite elements using the embedded discontinuities or XFEM. A smeared approach is more appropriate when describing a micro-crack formation process while a discontinuous one allows for a more realistic simulation of discrete macro-crack propagations. Usually, only one approach is used to simulate a fracture process in concrete during the entire deformation process. A combination of continuous and discontinuous approaches make it possible as in experiments to realistically capture all stages of fracture.

The coupled continuous-discontinuous constitutive law presented here is the extension of the model formulated in [1, 2]. The original model links a continuous and discontinuous description of cracks defined within continuum mechanics. In a continuum regime, two constitutive laws are used alternatively: an elasto-plastic one with a Rankine criterion and an isotropic damage one formulated within continuum

damage mechanics. Both laws are equipped with a characteristic length using an integral non-local theory to restore the well-posedness of the boundary value problem. Discontinuous displacement jumps are described with XFEM. An special algorithm is responsible for transferring a softening process from bulk points into a newly created crack segments. Discrete cracks are assumed to follow smeared crack patterns instead of using criteria based on a stress distribution (Fig. 1). The proposed extension introduces a transfer function which allows for a gradual switch from a continuous (smeared) to discontinuous (discrete) softening process. It is equal to zero at the tip (no discrete softening) and it increases along the crack length to the maximum value of 1 (no continuum softening). By extending a crack with a new segment, nodes and finite elements located in a band perpendicular to this segment are doubled. The width of a doubled zone covers the width of a localized zone. Both doubled element sets share the same nodes along zone boundaries. As a consequence no special algorithm is required to a force transfer of the softening mechanisms.



Fig. 1. Nooru-Mohamed test with shear force 10 kN: crack patterns and force-displacement curves for elasto-plastic law (left column), coupled model (middle column) and XFEM approach (right column) ('exp' - experiments)

- J. Bobiński, J. Tejchman. A coupled continuous-discontinuous approach to concrete elements. VIII International Conference on Fracture Mechanics of Concrete and Concrete Structures, FraMCoS-8, Toledo, Spain 2013.
- [2] J. Bobiński, J. Tejchman. A constitutive model for concrete based on continuum theory with non-local softening coupled with eXtended Finite Element Method. Computational Modelling of Concrete Structures, vol. I, 117-126, Taylor & Francis Group, 2014.

# The Thick Level Set approach, towards simulations coupling local and non-local evolutions.

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When simulating the fracture scenario of a structure using continuous local models, one often fall on a pathological mesh dependency of the approximate solution. The Thick Level Set approach (TLS) aims at writing models that don't suffer of such an issue. In this sense, the TLS shares some similarities with well-known approaches such as non-local integral approach [1] and gradient-enhanced approach [2].

The TLS was first proposed in [3] in a quasi static context and received further improvement of its discretization in [4], some analytical development in [5], an extension to explicit dynamics in [6] and a recent comparison in [7] with the well known phase field approach (in particular the model established in [8]). In these TLS related papers, modeling is limited to damage quasi-brittle behaviour. This limitation is due to the way the damage is confined in a bounded area called the damage band. This damage band as a  $2l_c$  thickness when crack emerges, where  $l_c$  is the characteristic length. Consequently, damage is only modeled in the fracture process zone. In this process zone, the damage treatment is purely non-local.

A recent extension of the TLS approach, proposed in [9], improves its generality by modeling diffuse damage away from the fracture process zone. As a result, the damage treatment is local and non-local: non-local in the fracture process zone, local away from it. The fracture process zone emerges or advances where and when diffuse degradations grow toward fracture. The key idea is to control damage field gradient by using a non-local treatment where and when damage gradient rises. To this end, an auxiliary field  $\phi$  is introduced, the damage field is deduced from it:  $d = d(\phi)$ . We further impose that  $\|\nabla \phi\| \le 1$  leading to the upper bound of the damage gradient  $\|\nabla d\| \le \frac{dd}{d\phi}(\phi)$ . The damage evolution is linked to  $\phi$  evolution.

In this work, we present simulations of the out-plane pull out test of a ring. This test case is treated in [9] as a unidimensional problem. We treat it as a bidimensional transverse displacement problem. We present a unified way to treat local and non-local zones based on the construction of two specific finite dimensional function spaces to approximate both  $\phi$  and its variation  $\Delta \phi$  during a time step (and related fields). The extended
finite element method is used as the underlying method to construct this two spaces. The propagation of the interface between local zone and non-local zone ensures the continuity of  $\phi$  (and thus the damage field).

- [1] Gilles Pijaudier-Cabot and Zdeněk Bažant. "Nonlocal damage theory". In: *Journal* of Engineering Mechanics 113.10 (1987), pp. 1512–1533.
- [2] R.H.J. Peerlings et al. "Gradient enhanced damage for quasi-brittle materials". In: *International Journal for Numerical Methods in Engineering* 39.19 (1996), pp. 3391– 3403.
- [3] Nicolas Moës et al. "A level set based model for damage growth: The thick level set approach". In: *International Journal for Numerical Methods in Engineering* 86.3 (2011), pp. 358–380.
- [4] Paul-Emile Bernard, Nicolas Moës, and Nicolas Chevaugeon. "Damage growth modeling using the Thick Level Set (TLS) approach: efficient and accurate discretization for quasi-static loadings". In: *Computer Methods in Applied Mechanics and Engineering* 233-236 (2011), pp. 11–27.
- [5] Claude Stolz and Nicolas Moës. "A new model of damage : a moving thick layer approach." In: *International Journal of Fracture* 174 (2012), pp. 49–60.
- [6] Kévin Moreau et al. "Explicit dynamics with a non-local damage model using the thick level set approach". In: *International Journal for Numerical Methods in Engineering* (Dec. 2014), n/a-n/a. ISSN: 00295981. DOI: 10.1002/nme.4824. URL: http://doi.wiley.com/10.1002/nme.4824.
- [7] Fabien Cazes and Nicolas Moës. "Comparison of a Phase-Field model and of a Thick Level Set model for brittle and quasi-brittle fracture". In: *International Journal for Numerical Methods in Engineering* (2014), pp. 1–31. DOI: 10.1002/nme.
- [8] Christian Miehe, Martina Hofacker, and Fabian Welschinger. "A phase field model for rate-independent crack propagation: Robust algorithmic implementation based on operator splits". In: *Computer Methods in Applied Mechanics and Engineering* 199.45-48 (Nov. 2010), pp. 2765–2778. ISSN: 00457825. DOI: 10.1016/j.cma. 2010.04.011. URL: http://linkinghub.elsevier.com/retrieve/pii/ S0045782510001283.
- [9] Nicolas Moës, Claude Stolz, and Nicolas Chevaugeon. "Coupling local and nonlocal damage evolutions with the Thick Level Set model". In: Advanced Modeling and Simulation in Engineering Sciences 1.1 (2014), p. 16. ISSN: 2213-7467. DOI: 10.1186/s40323-014-0016-2. URL: http://www.amses-journal.com/ content/1/1/16.

# Combined XFEM–continuous damage mechanics approach for three dimensional concrete crack propagation

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This work presents a numerical method to simulate three dimensional failure process of quasi-brittle concrete materials using a combination of continuous damage model (CDM) and extended finite element method (XFEM) [1].

Based on many experiments and numerical simulations at the level of particles, the mechanism of crack propagation in quasi-brittle materials can be idealized into four steps. In the first stage of loading the material behaves elastically; the loading and unloading behaviour is identical. During the second phase of the loading process, the formation and growth in the fracture process zone (FPZ) of densely distributed microcracks induces a non-linear response in the material. At this stage of the fracture process, the microcracks are not localized; the material remains stable and the load-deformation response no longer has a linear behaviour. After increasing the load beyond its peak value, coalescence and ramification of microscopic cracks, as well as the debonding of aggregates in the material matrix occurs and the response of the material at the macroscopic scale becomes unstable. Deformation and localization occur in a band of defined width, forming a weak discontinuity in the strain field. At this stage, the load-deformation response is in the softening branch. In the last stage, crack bridging occurs and the cohesive forces vanish which results in an area of strong discontinuity with a stress-free macrocrack.

The CDM approach can provide a relatively good model to describe the second stage of the idealized FPZ. However, in the final stage of failure, when damage localizes into a macroscopic crack, the continuum models are known to exhibit stress locking (spurious stress transfer), lack of objectivity (mesh-induced directional bias), and possible instability (appearance of spurious kinematic modes) [2]. In contrast, a cohesive XFEM crack model, does not describe the first phase of densely distributed microcracks very well. However, the macroscopic crack can be well-described with this approach. Consequently, combining CDM with XFEM in one single approach is an appealing technique to describe the failure process of quasi-brittle materials. Using this approach, the crack path can be predicted using the regularized local anisotropic continuum damage approach. The crack path orientation is corrected during loading using a rotative crack model. This approach is repeated until transition to XFEM occurs when the damage index exceeds a prescribed value. With energy conservation enforced for mode I energy dissipation during transition from CDM to XFEM, this approach was successfully applied in two-dimensional space [3] and is extended in three dimensional space in this work.

In 2D, the knowledge of the crack tip location in a single finite element and its propagation using the non-local principal stress direction guarantees satisfaction of the crack path continuity. However, for three dimensional simulations, the crack tip is replaced by a crack front consisting of several finite elements, which may locally make failure predictions that do not necessarily result in a continuous or a unique failure surface. The use of a global tracking algorithm [4], helps define a continuous and unique smooth, planar or curved crack surface at the cost of having to solve an additional global system of equations. After this additional equation is solved, the 3D crack path is defined from the isosurface extending the crack surface using a marching cube algorithm [5]. However this crack tracking algorithm can lead to non-planar crack surface within one finite element. For linear hexahedral elements, warped surfaces leads to special quadrature treatment for element integration.

This work presents the development of a three dimensional concrete cracking simulation strategy based on a transition from a continuum to a cohesive XFEM approach. We would like to share our experience on the numerous challenges that must be overcome to successfully model non planar crack propagation in three dimensional space. Seldom discussed the global tracking algorithm boundary conditions used to compute the crack path is introduced. Finally, numerous three dimensional test cases with non-planar crack surfaces are presented to perform verification and validation of the proposed model.

- [1] T. Belytschko and T. Black, "Elastic crack growth in finite elements with minimal remeshing," *International Journal for Numerical Methods in Engineering*, vol. 45, pp. 601-620, 1999.
- [2] M. Jirásek and T. Zimmermann, "Analysis of Rotating Crack Model," *Journal of Engineering Mechanics*, vol. 124, pp. 842-851, 1998.
- [3] S.-N. Roth, P. Léger, and A. Soulaïmani, "A combined XFEM–damage mechanics approach for concrete crack propagation," *Computer Methods in Applied Mechanics and Engineering*, vol. 283, pp. 923-955, 1/1/ 2015.
- [4] J. Oliver, A. E. Huespe, E. Samaniego, and E. W. V. Chaves, "Continuum approach to the numerical simulation of material failure in concrete," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 28, pp. 609-632, 2004.
- [5] W. E. Lorensen and H. E. Cline, "Marching cubes: A high resolution 3D surface construction algorithm," *ACM Siggraph Computer Graphics*, vol. 21, pp. 163-169, 1987.

# Experimental and theoretical investigations of concrete fatigue using damage approach with non-local softening

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Fatigue may be understand as the progressive and localized structural damage that occurs in materials under cyclic loading. In plain and reinforced concrete structures (e.g. bridges, highways, railway slabs, airport pavements, marine structures), fatigue causes excessive crack widths, debonding of reinforcement that finally leads to the structural failure. Our goal is to develop a realistic constitutive model to predict the fatigue life of plain and reinforced based on own experiments.

The comprehensive experiments were performed on plain concrete beams and concrete beams reinforced with steel and basalt bars (Fig.1). The cyclic load-displacement diagrams were registered for small (0.1-2 Hz) and medium frequencies (10-12 Hz). The fatigue life of beams was investigated with the cyclic stress rate varied between 0.60 and 0.95 of the static beams capacity. The development of fracture process zones and discrete macro-cracks was traced on the concrete surface by means of the DIC technique.

A numerical FE model was developed within continuum damage mechanics to describe the fatigue behaviour of concrete. The usual damage models [1] are not able to predict the fatigue strength since the damage growth does not occur if a subsequent reloading strain does not exceed the maximum previously experienced value. The extension from a static to fatigue formulation was carried out similarly to the formulation by Marigo [2]. The classical yield concept was replaced by a loading-unloading irreversibility concept. The assumption allowed for taking into account the damage accumulation for loading levels below the yield surface. Figure 2 shows the difference in the concrete response under cyclic loading conditions. For a damage formulation with the yield concept there is no a damage growth for subsequent loading and unloading levels below the yield surface. In turn, in a fatigue damage formulation, the cumulative damage growth under repeated cycles may be captured. The formulation was enhanced by a characteristic length of micro-structure by means of a non-local approach to properly describe strain localization. The model was calibrated based on the experiments with beams.



Fig.1. Fatigue diagram *S-N* from experiments with concrete elements including basalt reinforcement as compared with ACI recommendation



Fig.2. Force-displacement curves for concrete element in static and fatigue damage formulation for prescribed cyclic loading

- I. Marzec, J. Tejchman. Enhanced coupled elasto-plastic-damage models to describe concrete behaviour in cyclic laboratory tests: comparison and improvement. Arch. Mech., 64(3), 227–259, 2012.
- [2] J.J. Marigo. Modelling of brittle and fatigue damage for elastic material by growth of microvoids. Eng Fract Mech, 21(4), 861–74, 1985.

# Smeared damage model and crack localization for RC multi-fiber beam descriptions

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Based on the finding that damage is a major part of concrete behavior, a new damage model has been recently proposed (the " $\mu$  model" [1]) relying on the principles of isotropic damage mechanics. Two thermodynamic variables were defined in order to describe, within a 3D formulation, the unilateral behavior of concrete (i.e. crack opening and closure), which is essential for cyclic loadings and particularly the seismic response of concrete structures. Furthermore, the material parameters are easy to identify from individual tensile and compressive tests alone. A series of applications using classical FE calculations has yielded satisfactory results when compared to experimental results [2] thus attesting to the model's effectiveness.

To reduce the size of nonlinear problems for real structures, a simplified FE description has been considered for engineering purposes; it is based on the use of multi-fiber beam elements useful for frame structures (beams and columns).

#### Multi-fiber beams and strain localization

Strain localization is a major concern in maintaining the objectivity of finite element calculations. This problem is well known in classical 2D-3D calculations, for which two principal treatment options are available: i) the solution with material parameters adjusted to suit element size [3], and ii) implementation of regularization methods such as non-local methods [4]. Such a problem has never before been studied in the context of a multi-fiber beam description. More generally speaking, a fiber beam description contains localization if the behavior of the beam element displays softening. Localization is thus present for:

- a plain concrete element, regardless of its loading (tension or bending);
- a RC element if the loading is uniaxial and tensile (localization appears once the concrete has failed);
- a RC element in bending when the reinforcement ratio is less than a minimum value, referred to as the fragility ratio (see Eurocode 2).

For other reinforcement ratio values, if bending is dominant (should bending and tension be combined), no localization is present.

Hillerborg stipulated that the energy dissipated at failure in a unit concrete volume must be equal to the fracture energy. In the presence of localization, the control of result objectivity would then be the same as for classical 2D-3D FE calculations. Localization takes place within a band of elements, and the material parameters must be calibrated with the size h of these elements (Fig. 1a). In the absence of localization, the damagecracking processes for one crack are distributed on both sides of the crack over a volume defined by the distance  $s_c$  between two cracks (Fig. 1b). Therefore, the concept of crack band cannot be applied here. The calculation must be calibrated so that the fracture energy is consumed in a  $s_c$  wide area, leading us to write:



Fig. 1 Localization processes in a multi-fiber beam description: a) plain concrete: damage is localized on a band of elements (size h); and b) reinforced concrete: damage is distributed along the cracking zone (s<sub>c</sub> is the crack spacing)

This approach shows that the results obtained insured both, i/ to avoid dependence on mesh size as damage evolves and, ii/ to give results in good accordance with the 2D-FE description. In addition enhancements to take into account specific phenomena such as steel-concrete debonding, hysteretic loops and permanent strains on the concrete behavior have been included.

These concepts have helped build a tool for accessing with as good results as a conventional finite element calculation. Results include the global and the local levels (e.g. force-displacement, state of rebar deformation, average crack width) though with a better control of convergence problems and considerable computational time saving.

Along these same lines, other improvements are ongoing and include, i/ a description of the strain rate effect via a variation in the initial threshold of damage, so as to model high-velocity loading effects and, ii/ for simplified approaches based on the enhanced modeling presented herein, the development of equivalent lattice modeling for RC walls and the introduction of section warping to treat torsion and shear effects [5].

In conclusion, this model and the strategy proposed to achieve simplified modeling have provided a useful tool for engineering applications; moreover, this tool can cover a wide array of problems, whether monotonic or cyclic, encompassing quasi-static and dynamic loadings.

- [1] Mazars J., Hamon F., Grange S., 2014, A new 3D damage model for concrete under monotonic, cyclic and dynamic loading, Materials and Structures, 2014
- [2] Ragueneau F., Lebon G., Delaplace A., 2010, Analyse expérimentale du comportement cyclique de poutres en béton armé, LMT Internal report, October 2010.
- [3] Hillerborg A. et al., 1976, Analysis of crack formation and growth in concrete by means of fracture mechanics and finite elements, Cem. & Concrete Res., 6, 773-782.
- [4] Pijaudier-Cabot, G., and Bazant, Z. P. (1987), "Nonlocal Damage theory." J. Engrg. Mech., ASCE, 113(10), 1512-1533.
- [5] Capdevielle S. et al., 2014, Introduction of warping in a multifiber beam model: effect on the nonlinear analysis of R.C. structures under torsion, EURO-C, Austria.

# **3D** ductile crack propagation with the XFEM

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In industrial forming processes metals are subjected to large plastic deformations. Related to the plastic deformation pores will start to grow, merge and finally lead to micro cracks within the material. Macroscopically this is recognized as a loss of stiffness of the structural behavior. This effect is usually modeled in the context of damage mechanics, where in the last fifteen years several theories were established to circumvent the mesh dependent localization. Most of them are based on the introduction of a new degree of freedom for the thermodynamic driving force of the damage. This new degree of freedom is then computed by solving an additional scalar balance equation of *Helmholtz* type. Nevertheless, if damage evolves and is used to predict macro cracks the global stiffness matrix becomes ill conditioned. Furthermore cracks are just represented in a smeared way and their dimension is related to the mesh size and some artificial internal length parameter. To overcome this drawback the damage is transfered to discrete cracks if it exceeds a material depended threshold value. The representation of fracture is modeled with the eXtended Finite Element Method (XFEM). This numerical tool enables a nearly mesh independent crack representation. The strong discontinuities are modeled by level set functions and the spatial discretization of the displacement field becomes enriched. The enrichment functions are chosen in a way to enable discontinuities in the displacement and even singularities or nearly singularities in derived quantities. Within the XFEM only a local neighborhood of the crack becomes enriched and the overall number of unknowns increases only moderately.

Classical crack propagation criteria loose their validity in a finite deformation context and inelastic material behavior. The argumentation for the chosen damage based criterion is that once the crack is initiated it will propagate if the pores around the crack front will continue to coalesce. So the mechanism for crack initiation and propagation are the same. The problem of evolving the front is treated with a purely geometric approach, which preserves the property of perpendicular level sets. After the propagation no remeshing is needed, but the enrichment scheme has to be updated in the vicinity of the crack front. There is also a need for adapting the history variables in these elements. This becomes necessary because the integration scheme alters with the enrichments.

The load displacement path of cracking structures is highly challenging. That's the reason why neither an entirely force driven nor a displacement driven simulation is able to follow the path and an arc length approach is used.

Keywords: XFEM,crack propagation, non local damage

# Nucleation, initiation, and propagation of cracks in gradient damage models

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Damage models can be regarded as a variational regularisation of brittle fracture. The energy of a cracked solid with linear elasticity and Griffith surface energy can be approximated in the sense of Gamma-Convergence by damage models with gradient on the damage variable. The damage models are characterised by a total energy function in the following form:

$$\mathcal{E}_{\ell}(u,\alpha) = \int_{\Omega} \frac{1}{2} a(\alpha) A_0 \epsilon(u) \cdot \epsilon(u) \mathrm{d}x + \frac{G_c}{c_w} \int_{\Omega} \left( \frac{w(\alpha)}{\ell} + \ell \,\nabla \alpha \cdot \nabla \alpha \right) \mathrm{d}x, \qquad (1)$$

where u(x) is the displacement,  $\alpha(x) \in [0, 1]$  the damage field,  $\epsilon(u)$  the linear deformation measure,  $A_0$  the elastic stiffness tensor,  $G_c$  the fracture toughness,  $c_w$  a normalisation constant. The function a et w identify the damage constitutive law, representing stiffness modulation and energy dissipation. The key parameter in this model is the *internal length*  $\ell$ . Gamma-convergence shows that for a large class of a and w, the global minima of the energy of the damage model converge toward the global minima of the energy of the Griffith fracture model. However, evolutions obtained with a local minimisation criterion on the damage energy strongly depend on  $\ell$ , and may not converge toward the evolutions obtained with (local or global) minimisation criteria on the Griffith model. Gradient damage models introduce pertinent physical effects, as size effects and maximum allowable stress, that are missing in the Griffith model.

In this contribution we show how the choice of the constitutive function a and w and the internal length  $\ell$  may influence (or not) the evolutions based on local minimisation criteria on (1). The results will be illustrated mainly through carefully selected numerical experiments verified against analytical or semi-analytical results.

We will consider the following problems:

- The *nucleation* of a crack from a structure without stress singularity (e.g. a homogenous bar or a bar with a hole).
- The *propagation* of preexisting cracks along straight or curved paths.
- The *initiation* of a crack in a structure with a stress singularity represented by a notch with a finite opening angle.

# Ductile damage modeling with locking-free regularized GTN model

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The study of damage development in industrial structures has become essential to avoid safety risks and economic losses. In complement to experimental investigations, advanced numerical simulation of damage appears as a promising approach. However, damage modeling still faces several difficulties. In the case of ductile damage, characterized by a large amount of plastic strain and negative hardening, plastic volumetric locking and spurious strain localization are observed and should be dealt with.

In this work, a logarithmic finite strain formulation is used which allows expressing the constitutive equations as in the small-strain theory. The damage model is based on the Gurson-Tvergaard-Needleman (GTN)<sup>[1]</sup> model which introduces an internal variable describing the evolution of void volume fraction in the material. The complete model takes into account the three steps of the damage evolution (i) cavity nucleation, (ii) cavity growth and (iii) cavity coalescence<sup>[2]</sup>. A non-local model with regularization of the equivalent plastic strain<sup>[3]</sup> is applied to the GTN criterion so as to solve mesh dependency and strain localization problem. Parametric studies are used to establish a simplified relation between the width of the localization band and the non-local regularizing parameter. Finally, the non-local model is combined with mixed "displacement-pressure-volume variation" elements to avoid volumetric locking.

The proposed model is used to simulate normalized specimens (NT, CT) with good agreement with experimental results obtained for a piping steel. Based on these preliminary tests, a simulation on a real-dimension tube will be performed.

- V. Tvergaard, A. Needleman. Analysis of the cup-cone fracture in a round tensile bar. Acta Metall., 32, 157–169, 1984.
- [2] J. Besson, D. Steglich, W. Brocks, Modeling of plane strain ductile rupture. International Journal of Plasticity 19 (1517–1541), 2003.
- [3] E. Lorentz, V. Godard. Gradient damage models: Toward full-scale computations. Comput. Methods Appl. Mech. Engrg, 1927-1944, 2011.

# **Record breaking events in crackling time series**

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We investigate the statistics of record breaking (RB) events in the time series of crackling bursts in a fiber bundle model of the creep rupture of heterogeneous materials. In the model fibers break due to two mechanisms: slowly accumulating damage triggers bursts of immediate breakings analogous to acoustic emissions accompanying fracture. Record events are defined as bursts which have a larger size than all previous events in the time series. We analyze the statistics of records both in the limit of equal (ELS) and local load sharing (LLS) of the model and compare the results to the record statistics of sequences of independent identically distributed random variables [1, 2, 3, 4, 5].

For ELS (mean field limit), the number of records was found to grow with the logarithm of the event number except for the close vicinity of macroscopic failure where an exponential dependence is evidenced. The two regimes can be attributed to the dominance of disorder with small burst sizes and to stress enhancements giving rise to efficient triggering of extended bursts, respectively. Both the size of records and the increments between consecutive record events are characterized by power law distributions with a common exponent 1.33. In case of the waiting time distributions we found two distinct exponents for low and high loads. Studying the evolution of records we identify a load dependent characteristic scale of the system which separates slow down and acceleration of RB as failure is approached. When the interaction of fibers is localized RB sequences show qualitatively the same behaviour, however, with significantly different exponents [3].

- [1] Z. Halász, Zs. Danku and F. Kun, Phys. Rev. E 85, 016116 (2012).
- [2] Zs. Danku and F. Kun, Phys. Rev. Lett. 111, 084302 (2013).
- [3] S. Lennartz-Sassinek, I.G. Main, Zs. Danku and F. Kun, Phys. Rev. E 88, 032802 (2013).
- [4] Zs. Danku and F. Kun, Sci. Rep. 3, 2688 (2013).
- [5] Zs. Danku and F. Kun, Front. Physics 2, 8 (2014).

# Strain localisation from eikonal nonlocal damage formulation with evolving internal length

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Integral nonlocal techniques are often used to regularise damage models in the presence of strain softening laws. Some questions concerning the identification of the internal length, its possible evolution during damage process and the need for special treatments of non-locality operators near the boundaries (e.g. edges, cracks) are however still open. A physical request is that material points separated by a crack or highly damaged zones should not interact (despite what is done in standard nonlocal integral theories).

This can be obtained by introducing an evolving internal length [1, 2, 3, 4] depending on mechanical fields (e.g. damage, strain, stress, ...), thus representing a progressive transition from Continuum Damage Mechanics to Fracture Mechanics when crack localises.

A novel interpretation of evolving nonlocal interactions was recently introduced by [5]. Based on the Wentzel-Kramers-Brillouin approximation for high-frequency wave propagation in a damaged medium, the authors defined the interaction distances as the solution of an Eikonal equation (i.e. a stationary case of the Hamilton-Jacobi equation). If a differential geometry interpretation is adopted, this leads to consider that damage curves the space in which the interaction distances are computed. Consequently, nonlocal interaction distances between each pair of material points coincide with their geodesic distances (i.e. the shortest paths in a curved Riemannian space).

The present work investigates and discusses numerical implementation and properties of such a regularisation technique, when applied to two-dimensional isotropic damage modelling.

Numerical developments are performed in Python language through the PyFEM nonlinear finite element code [6]. An object-oriented Finite Elements (FE) implementation of standard integral nonlocal damage is first developed and discussed. The numerical formulation is then enhanced by introducing evolving non-local interactions. Eikonal equation is numerically solved over a conveniently defined computational grid (i.e. a connected, undirected and weighted graph) using a Fast Marching Method [7, 8].

Two-dimensional structured and unstructured FE meshes composed of 3-noded linear triangular elements are considered in the paper. For this particular problem a Fast Marching grid can be simply defined based on the FE mesh. Graph's vertices are assumed to coincide with the elementary centres of gravity (i.e. gauss points), while graph's arcs are

defined by all the segments relying each pair of vertex disposed on two opposite sides of each elementary edge (FE mesh). The initial weight (in undamaged condition) of each arc of the graph is assumed to coincide with the cartesian distance between its extremities. The influence of material damage is then taken into account by allowing the weight of the edges to evolve (increase) depending on the damage variables pertaining to the connected vertices.

Once this schematisation is introduced, a first-order accurate Fast Marching algorithm is used (at each mechanical loading step) to compute geodesic distances, and by that to extract the shortest path in Riemannian space curved by damage between each pair of interacting points. These computed distances are finally used, in FE computations, to define the kernel of Gaussian weighting functions to be used in integral nonlocal regularisation.

Several numerical examples are considered to analyse the proposed method. The influence of one or more damaged elements on nonlocal interactions is analysed for both structured and unstructured FE meshes. Finally, strain localisation in a 2D bar submitted to a uniaxial traction is numerically simulated.

- [1] Pijaudier-Cabot, G., Haidar, K., Dubé, J.-F., 2004. Non-local damage model with evolving internal length. *International Journal for Numerical and Analytical Methods in Geomechanics*, 28 (7-8), 633-652.
- [2] Desmorat, R., Gatuingt, F., 2007. Introduction of an internal time in nonlocal integral theories, *Internal report LMT-Cachan*, number 268, 2007 <a href="http://hal.archivesouvertes.fr/">http://hal.archivesouvertes.fr/</a> hal-00200898/en/>.
- [3] C. Giry, F. Dufour, J. Mazars, Stress-based nonlocal damage model. *Int. J. Solids and Structures*, 48(25-26), 3431-3443, 2011
- [4] D. Grégoire, L.B. Rojas-Solano, G. Pijaudier-Cabot, Continuum to discontinuum transition during failure in non-local damage models, *International Journal for Multiscale Computational Engineering*, 10 (6) (2012) 136.
- [5] R. Desmorat, F. Gatuingt, M. Jirasek. Nonlocal models with damage dependent interactions motivated by internal time *Engineering Fracture Mechanics (submitted)*, 2015.
- [6] R. De Borst, M.A. Crisfield, J.J.C. Remmers, C.V. Verhoosel, Nonlinear Finite Element Analysis of Solids and Structures, 2nd Edition, Wiley, 2012.
- [7] J.A. Sethian. A fast marching level set method for monotonically advancing fronts. *Proc. Nat. Acad. Sci.*, 93, 1996.
- [8] R. Kimmel, J.A. Sethian. Computing Geodesic Paths in Manifolds Proc. Nat. Acad. Sci., 95, 8341-8435, 1998.

# Isogeometric Implementation of the High-Order Microplane Model for Softening and Localization

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In this study, a recently developed higher order microplane (HOM) model for softening and localization [1], is numerical implemented within a finite elelemnt framework based on isogeometric analysis. The HOM model was derived directly from a three dimensional discrete particle model and it was shown to be associated with a high order continuum chracterized by independent rotation and displacement fields. Furthermore HOM model was demonstrated to possess two characteristic lengths: the first associated with the spacing of flaws in the material internal structure and related to the gradient character of the continuum; and the second associated with the size of these flaws and related to the micropolar character of the continuum. The displacement-based finite element implementation of this type of continua requires C1-continuity both within the elements and at the element boundaries. This motivated the implementation of the concept of isogeometric analysis which ensuress a higher degree of smoothness and continuity [2]. NURBS based isogeometric elements were implemented in a 3D setting and with both displecement and rotational degrees of freedom at each control point. The performed numerical analyses demonstrate the effectivness of the proposed HOM model implementation to ensure optimal convergence in both elastic and softening regime. Particualry, the proposed approach prevents strain localization and spurious mesh sensitivity known to be patological issues for typical local strain-softening constitutive equations.

- [1] Cusatis, G., Zhou, X. High-order microplane theory for quasi-brittle materials with multiple characteristic lengths. *Journal of Engineering Mechanics*, 140(7), 2013.
- [2] Hughes, T. J., Cottrell, J. A., Bazilevs, Y. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer methods in applied mechanics and engineering*, 194(39), 4135-4195, 2005.

# Remarks on constitutive laws and influence functions used in the Peridynamic theory

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Peridynamics is a recently proposed continuum theory based on a non local approach and formulated with integral equations [1-3]. This theory is particularly suitable for dealing with crack propagation in solid materials. Since the presence of cracks in a solid represents a discontinuity in the displacement field, the peridynamic formulation, which uses only integrals, overcomes the problems of the classical continuum theory of solid mechanics. In this way, the theory is able to treat the spontaneous formation of discontinuities at different locations together with their mutual interaction and growth in a consistent framework without any ad hoc criteria.

The main assumption of Peridynamics is that the body is composed of material points which interact with other points within a finite distance  $\delta$  named horizon, which is linked to the characteristic length scale of the material and/or of the considered phenomenon [4]. In its simplest formulation, the equation that controls the motion of the points of the structure assumes the following form:

 $\rho \ddot{\mathbf{u}}_{i} = \int_{H_{i}} \mathbf{f} \Big[ \mathbf{u}(\mathbf{x}_{j}, t) - \mathbf{u}(\mathbf{x}_{i}, t), \mathbf{x}_{j} - \mathbf{x}_{i} \Big] dV_{j} + \mathbf{b}(\mathbf{x}_{i}, t) \qquad \forall \mathbf{x}_{j} \in H_{i}$ 

where  $H_i$  is the spherical region of radius  $\delta$  centered at point  $x_i$ , u is the displacement vector field, b is a body density force vector and f is called pairwise force function, which is a force per unit volume squared that represents the force that point  $x_j$  exerts on point  $x_i$  (*bond*). All constitutive information is contained in the function f. Introducing vectors  $\xi$  and  $\eta$  respectively as the initial relative position and the current relative displacement of the points  $x_i$  and  $x_j$ , the most common definition of the force function f is:

$$\mathbf{f} = c \cdot s \cdot \mu(\boldsymbol{\xi}) \cdot \frac{\boldsymbol{\eta} + \boldsymbol{\xi}}{\left| \boldsymbol{\eta} + \boldsymbol{\xi} \right|}$$

where *c* is the bond stiffness (generally called micromodulus),  $\mu(\xi)$  is a historydependent scalar function which takes the values 1 or 0 depending on the bond status (intact or broken) and *s* is the bond stretch. Considerations regarding the comparison between the elastic deformation energy in the classical theory of continuum and in Peridynamics [1] provide the definition of the bond stiffness *c* in terms of the Young's modulus E and of the horizon radius  $\delta$ . In the same way, the energy release rate of the material is related to the energy needed to break all the bonds crossing the fracture surface [6]. The micromodulus c can be supposed dependent [5] on the initial relative position introducing an *influence function* to weight every bond inside the horizon. In [7], it is demonstrated that even simple generalizations of these influence functions allow for the modulation of nonlocal effects within a constitutive model in order to describe a rich range of dynamic behaviour: the authors concentrate their attention on dispersive elastic wave correlation with a family of influence functions using the linear prototype microelastic brittle (PMB) constitutive laws.

In this paper the effect of influence functions and a wider class of constitutive laws (linear, bi-linear and tri-linear) will be investigated in relation to the crack paths obtained for static and dynamic problems. Then, the fracture energy release for the various cases and the distribution of fracture energy density inside the horizon will be evaluated: in fact the fracture energy density inside the horizon is variable even for the simplest constitutive law, i.e. PMB with unitary influence functions (no modulation of nonlocal effects). The effects of the influence functions on the fracture energy density distribution will be studied choosing constitutive laws that require the same energy to break a single bond.

- [1] S.A. Silling. Reformulation of elasticity theory for discontinuities and long-range forces. Journal of the Mechanics and Physics of Solids, 48(1), 175-209, 2000.
- [2] S.A. Silling, M. Epton, O. Weckner, J. Xu, E. Askari. Peridynamic States and Constitutive Modeling. Journal of Elasticity, 88, 151–184, 2007.
- [3] S.A. Silling, R.B. Leouchq. Peridynamic Theory of Solid Mechanics, Advances in Applied Mechanics, 44, 73-168, 2010.
- [4] F. Bobaru and W. Hu. The meaning, selection, and use of the peridynamic horizon and its relation to crack branching in brittle materials, International Journal of Fracture, 176, 215-222, 2012.
- [5] Y. D. Ha, F. Bobaru. Studies of dynamic crack propagation and crack branching with peridynamics. International Journal of Fracture, 162(1-2), 229-244, 2010.
- [6] S.A. Silling, E. Askari. A meshfree method based on the peridynamic model of solid mechanics. Computational Structures, 83, 1526–2535, 2005.
- [7] P. Seleson, M. Parks. On the role of the influence function in the peridynamic theory. International Journal of Multiscale Computational Engineering, 9(6), 689-706, 2011.

# A non-local damage model for ductile fracture

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Ductile fracture may be modelled accurately utilising a unified approach combining Continuum Damage Mechanics and Fracture Mechanics concepts. Continuum theories may describe adequately large plastic deformations in ductile materials that may lead to internal degradation phenomena but near to failure a discontinuous model may be more suitable to represent surface decohesion and macro-crack propagation.

A possible solution to build a numerical model including the best features of the two approaches is to combine a FEM continuous damage model with the XFEM to deal with macro-cracks and their evolution. The continuous model is used to determine damage evolution and localization. Then from a critical damage state a macro-crack is introduced.

Several problems must be dealt with, so that a robust model is obtained. Firstly, during damage evolution and consequent local softening the traditional mesh dependence problem must be addressed. Here an integral non-local approach, associated with the damage variable, is used to regularize the numerical solution. Then a criterion for the transition from a full continuous to a partial discontinuous solution must be defined. A criterion based on critical damage value is adopted and an algorithm to avoid instabilities near to fracture is included. The algorithm is based on the inclusion of a cohesive law in the interaction between the two surfaces of a crack to model the material which is not fully degraded, so that an energetically consistent transition from damage to fracture can be obtained. Geometrical and energetic criteria are used to define crack size and orientation. Special attention is given to numerical integration within elements which are partially or totally cracked, having in view the incompressibility of plastic deformation associated with ductile behaviour. The subsequent crack propagation is governed by damage evolution near the crack tip.

### References

 Mariana R. R. Seabra, Primoz Sustaric, Jose M. A. Cesar de Sa, Tomaz Rodic. Damage driven crack initiation and propagation in ductile metals using XFEM. Comput Mech (2013) 52:161–179.

# Nonlocal Averaging Near Boundaries and Its Influence on Size Effect

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Regularized formulations of constitutive models with softening can restore well-posedness of the resulting boundary value problem and provide numerical solutions that properly reflect localized dissipative processes in the material microstructure. All such formulations introduce (directly or indirectly) a new parameter with the dimension of length, which corresponds to the intrinsic length of the material and controls the width of the localized process zone and thus also the dissipated energy. For integral-type nonlocal formulations, such parameter is typically contained in the definition of the kernel of the averaging integral. Traditional nonlocal formulations usually assume that, in an infinite medium, the strength of the nonlocal interaction between two points depends exclusively on their distance. This simplifying assumption must be abandoned near the physical boundary of a finite body of interest, and a modified averaging scheme is needed in order to avoid or at least reduce various pathological boundary effects. Some refined models modify the averaging operator even a points far from the boundary and, by considering a variable intrinsic length in an evolving microstructure, attempt a better description of the transition from damage concentrated in a narrow but finite band to fully localized fracture.

The present study is a continuation and expansion of previous work [1], in which the distribution of dissipated energy density near convex and non-convex boundaries was investigated and the numerical results obtained for various nonlocal damage formulations were compared to the results of analyses based on a meso-mechanical model. Here, the objective is to demonstrate the link between the precise form of nonlocal averaging near the boundary and the size effect on nominal strength. The nonlocal formulations considered here include the standard rescaling of the weight function, an averaging scheme based on the so-called local complement (initially proposed in [2]), an approach modifying the internal length depending on the distance from the boundary, and an anisotropic averaging that takes into account the stress state. Based on numerical simulations, the size effect on nominal strength is evaluated for geometrically similar beams with notches of different relative depths, including unnotched beams. The numerical results are compared to a recently published large set of experimental data acquired for plain concrete specimens from the same batch [3]. It is shown that, with the standard rescaling of the weight function, it is impossible to achieve a good overall agreement for the whole range

of specimen sizes and geometries. Acceptable results can be obtained with the modified approaches, especially with the distance-based formulation, which is relatively simple and efficient yet leads to realistic results. This reinforces the conclusions of [1], which were based on comparisons against a meso-scale model and did not take into account any experimental data.

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- [1] Grassl, P., Xenos, D., Jirásek, M. and Horák, M. Evaluation of nonlocal approaches for modelling fracture near nonconvex boundaries. *International Journal of Solids and Structures*, vol. 51, number 18, 3239–3251, 2014.
- [2] Borino, G., Failla, B. and Parrinello, F. A symmetric nonlocal damage theory. *International Journal of Solids and Structures*, vol. 40, number 13, 3621–3645, 2003.
- [3] Hoover, C.G., Bažant, Z.P., Vorel, J., Wendner, R., and Hubler, M.H. Comprehensive concrete fracture tests: Description and results. *Engrg. Fracture Mechanics*, vol. 114, 92–103, 2013.

# Isotropic and anisotropic damage-dependent interactions motivated by internal time

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The effect and the formulation of boundary conditions – such as free edges, notches and initial cracks – remain an open question for nonlocal models. One of the drawbacks of the standard nonlocal (NL) integral theory [1] consists in the nonphysical interaction, through the nonlocal averaging process, of points across a crack or a hole. The definition of natural boundary conditions of vanishing normal derivative of the nonlocal variable at the boundary is still under discussion for implicit gradient formulations. The continuous nucleation of a crack of zero thickness is not so simple as the thicknesses of the localization band and also of the band in which damage tends to 1 are not zero. They are proportional to the internal length introduced in the nonlocal approach. Local behavior along free edges – i.e. with a vanishing internal length – has been obtained by some authors [2-5]. The consideration of an internal length evolving with either the damage, the strain or the stress [6-9] seems a way to properly bridge Continuum Damage Mechanics and Fracture Mechanics as the internal length may then vanish for large values of damage. Such a feature is nevertheless quite difficult to enforce in nonlocal integral theories.

One attempts here to propose a solution to these main difficulties, first within the framework of the integral theory. The initial idea [10-11] is to keep the nonlocal integral averaging process but to quantify the distance between points as an effective distance, i.e. as a distance function for instance of the geometry and the state of the material between interacting points. In nonlocal internal time theory such an effective distance is defined from a dynamic process: how information (wave) propagates between interacting points. An internal time, constant, is introduced instead of an internal length, measured as evolving. Wave propagation provides information on interacting / non interacting points for the definition of the nonlocal averaging. The proposed approach naturally applies to both isotropic and anisotropic damage.

The question of the numerical implementation with damage/softening models naturally arises and is addressed here in 1D in order to point out the properties of the modified (wave) nonlocal approach. An important result of the 1D bar analysis performed is that mesh independency can be obtained with damage approaching 1 at a single point (and not in a spreading zone).

The pure geometric point of view that the effective distance between interacting points is increased by the damage field between them is then extended to 3D by means of the differential geometry interpretation that damage, possibly anisotropic, curves the space. Wentzel-Kramers-Brillouin (WKB) approximation for high-frequency wave propagation in a damaged medium allows us to define effective (interaction) distances as the solution of an eikonal equation, either isotropic or anisotropic, depending on the nature, scalar or tensorial, of the damage variable. This

builds a 3D pure geometric approach with no need anymore of many additional dynamics computations in a FE analysis. Such a result will simplify the numerical implementation in 2D or 3D as efficient algorithms for solving the eikonal equation (e.g. Fast Marching Methods) are available for both the isotropic and the anisotropic cases.

The proposed pure geometric approach applies to nonlocal integral as well as gradient-enhanced theories, making in both frameworks the internal lengths evolve and defining Eikonal NonLocal (ENL) damage models.

- [1] Pijaudier-Cabot G., Bažant Z., Nonlocal damage theory, *J. Engng Mech.*, ASCE, 113: 1512–33, 1987.
- [2] Pijaudier-Cabot G., Krayani A., Dufour F., Comments on boundary effects in non local damage based models, chapter in Nonlocal Modeling of Materials Failure, H. Yuan and F.H. Wittmann Ed., Aedificio Pubs, 2007.
- [3] Krayani A., Pijaudier-Cabot G., Dufour F., Boundary effect on weight function in nonlocal damage model, *Engineering Fracture Mechanics*, 76(14): 2217-2231, 2009.
- [4] Pijaudier-Cabot G., Dufour F., Non local damage model: boundary and evolving boundary effects, *European Journal of Environmental and Civil Engineering*, 14, 6-7, pp. 729-749, 2010.
- [5] Bažant, Z., Le J., Hoover C., Nonlocal Boundary Layer (NBL) model: overcoming boundary condition problems in strength statistics and fracture analysis of quasibrittle materials. In: Proc., FraMCoS-7, Korea, pp. 135-143, 2010.
  - [6] Geers M., de Borst R., Brekelmans W., Peerlings R., Strain-based transient-gradient damage model for failure analyses, *Comput. Methods. Appl. Mech. Engng*, 160: 133-153, 1998.
- [7] Pijaudier-Cabot, G., Haidar, K., Dubé, J.-F.. Non-local damage model with evolving internal length. *International Journal for Numerical and Analytical Methods in Geomechanics*, 28 (7-8): 633-652, 2004.
- [8] Simone A., Wells G.N., Sluys L.J., From continuous to discontinuous failure in a gradient enhanced continuum damage model, *Comput. Methods Appl. Mech. Engng*, 192(41-42): 4581-4607, 2004.
- [9] Nguyen G.D., A damage model with evolving nonlocal interactions, *Int. J. Solids & Structures*, 48, 1544-1559, 2011.
- [10] Desmorat, R., Gatuingt, F., 2007. Introduction of an internal time in nonlocal integral theories, *Internal report LMT-Cachan*, number 268, 2007 <a href="http://hal.archivesouvertes.fr/">http://hal.archivesouvertes.fr/</a> hal-00200898/en/>.
- [11] Desmorat, R., Gatuingt, F., Jirásek, M., Nonlocal models with damage dependent interactions motivated by internal time, *Engineering Fracture Mechanics (submitted)*, 2015.

# Nonlocal Regularization for Loss of Ellipticity in Inelastic Problems

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A mixed three-field finite element formulation is proposed as a method for the alleviation of mesh dependency in finite-deformation inelastic problems subjected to bifurcation. The additional fields in the formulation are the internal variables that cause bifurcation and the corresponding conjugate thermodynamic forces. The formulation leads to expressions for the additional fields that are akin to nonlocal averages that extend to a domain defined by the interpolation functions selected for these fields. The size of the nonlocal domain controls the length scale of the problem. The performance of the formulation is demonstrated by several examples where it is shown that it is effective in alleviating mesh dependence.

The formulation provides effective regularization, entirely by-passes the introduction of new and independent mechanical variables into the problem, discretizes naturally, computes averages with regular interpolation functions, and does not require changes to constitutive relationships.

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# Relationships between the cohesive zone model (CZM) and thick level set (TLS) damage model for quasi-brittle fracture

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Two main families of methods exist to model failure of quasi-brittle structures. The first one consists on crack based models, like cohesive zone models. The second one is a continuum damage approach that leads to a local loss of stiffness. Local damage models need some regularization in order to avoid spurious localization. A recent one, the Thick Level Set damage model, bridges both families by using level-sets. Cohesive and TLS models are presented. The cohesive one represents quasi-brittle behaviors with good accuracy but requires extra equations to determine the crack path. The TLS has proved its capability to model complex crack paths while easily representing cracks (i.e. displacement jumps); contrary to most damage models.

A one-dimensional analytical relationship is exhibited between TLS and cohesive models. The local damage behavior needed to obtain the same global behavior of a bar than with cohesive model is derived. It depends on the choice of some TLS parameters, notably the characteristic length lc. This local behavior is applied to bi-dimensional simulations of three point bending as well as mixed-mode single edge notched specimens are performed. Results are compared to cohesive simulations, regarding both crack paths and force-CMOD curves.

Force-CMOD curves obtained are very similar with both models. Theoretical analysis in 1D and numerical results in 2D indicates that, as lc goes to zero, TLS results tend to CZM ones.

The TLS model yields very similar results to the cohesive one, without the need for extra equations to determine the crack path.

# A thermodynamically consistent framework to couple microplane damage and plasticity models

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Inelastic phenomena in the mechanical performance of solids are mainly attributed to two distinct irreversible and energy-dissipative processes: (1) plastic deformation, that takes place along the slip planes, and (2) damage, which generally is caused by the initiation, coalescence and propagation of microcracks and microvoids at regions of high stress concentrations. The development of theoretical and posterior computational models that describe such complex behavior has received a great deal of attention as a consequence of their importance in practical engineering applications. These models find their theoretical foundations in Plasticity theories, Continuum Damage (CDM) and Fracture Mechanics (FM), whereas their corresponding numerical models have been mostly developed within the context of FEM.

For a wide variety of quasi-brittle materials, the constitutive microplane models of damage constitute a modeling strategy that can accurately describe the anisotropic development and growth of microcracks as well as plastic effects [3]. For the particular case of damage, in comparison to tensorial-like damage formulations [1], microplane-based models provides a simple and efficient way for representing the shift from isotropy to anisotropy. Through the use of such modeling strategy, the inelastic deformation, e.g. damage and/or plasticity, is linked with its orientation and the material response is directly evaluated for this orientation. Therefore, the material response can be evaluated by means of projecting the macroscopic stress and strain tensors in different planes of given orientations, which are denominated as microplanes. Based on this projecting procedure, the overall response of the material can be obtained through the combination (average) of the responses corresponding to all possible orientations [3,4].

This contribution presents a theoretical formulation of a thermodynamically consistent framework to couple microplane damage and plasticity models. In this setting, a first attempt to coupled damaged-plasticity microplane-based models was developed in [4], preserving the thermodynamic consistency of the formulation following an analogous scheme to that formerly developed for macroscopic plasticity-damage models based on CDM, see [5]. Differing from former alternatives and in analogy to the formulation of classical macroscopic coupled models given in [2], a different strategy is followed to formulate such thermodynamically consistent framework to perform the coupling procedure between both dissipative mechanisms. In this way, the resulting formulation herein proposed furnishes a coupling scheme without relying on the classical assumption such that the first of these inelastic phenomena induces the subsequent one (and therefore the computation order among them is fixed prior to the start of the computation). In contrast to this, we present a theoretical framework and its corresponding computational derivation in which none of the inelastic mechanisms has prevalence over the other one. Moreover, the term consistent here refers to two main aspects with regard to the present formulation: (1) the satisfaction of the second law of the thermodynamics through the evaluation of the Clausius-Duhem inequality, and (2) the fact that the original damage and plasticity microplane based models remain unmodified throughout the envisaged coupling procedure.

The modeling strategy herein developed can be used as a basic framework for the development of computational models for the quasi-brittle materials when damage and plasticity appear together. Thus, from the computational point of view, a detailed description of the numerical algorithm used for the computational implementation is outlined. The corresponding numerical model was incorporated into the general purpose finite element code FEAP by means of a user-defined material routine. Finally, a simple numerical example is presented to examine the capability of the framework to capture both inelastic mechanisms.

- [1] Abu Al-Rub, R.K., Voyiadjis, G.Z. On the coupling of anisotropic damage and plasticity models for ductile materials. *International Journal of Solids and Structures*, 40, 2611-2643, 2003.
- [2] Ibrahimbegovic A., Markovic D., Gatuingt F. Constitutive model of coupled damageplasticity and its finite element implementation. *European Journal of Computational Mechanics*, 12, 381-405, 2003.
- [3] Kuhl E., Carol I., Steinmann P. New thermodynamic approach to microplane model. Part II: Dissipation and inelastic constitutive modelling. *International Journal of Solids and Structures*, 38, 2933-2952, 2001.
- [4] Leukart M., Ramm E. A comparison of damage models formulated on different material scales. *Computational Materials Science*, 28, 749-762, 2003.
- [5] Simo J.C., Ju W. Stress and strain based continuum damage model-i. formulation, ii. computational aspects. *International Journal of Solids and Structures*, 23, 821-869, 1987.

# Benefits of numerical experimentation to formulate and calibrate constitutive laws for concrete

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The formulation and use of robust macroscopic constitutive laws that can handle the complex behaviour of quasi-brittle materials, such as concrete, is still an open question [1]. Despite the fact that significant progresses have been noticed over the last decades, further efforts are still needed when dealing with the case of cyclic loadings.

One of the main limitations is the lack of experimental data required to characterize and to identify these complex constitutive laws, which need numerous material parameters. In this work, we propose to use a fine scale model to provide reference data instead. The fine scale model lies in a combination of a lattice and rigid particles (DEM), of which, the behaviour, the identification process and the efficiency have been investigated previously [2, 3].

First, the fine scale model is used to simulate the concrete behaviour under uni-axial tensile cyclic loading. From original post-treatments (number of closed-cracks, frictional dissipated energy), a new and robust formulation of constitutive laws is identified. Second, from the variations of different energies (elastic, fracture, friction) in the fine scale model, the full set of material parameters related to the macroscopic model is calibrated. Representative simulations at the member scale are carried out for validation purposes.

- Richard, Benjamin, and Frederic Ragueneau. "Continuum damage mechanics based model for quasi brittle materials subjected to cyclic loadings: Formulation, numerical implementation and applications." Engineering Fracture Mechanics 98 (2013): 383-406.
- [2] Vassaux, Maxime, Richard, Benjamin, Ragueneau, Frederic, Millard, Alain. "Lattice models applied to cyclic behavior description of quasi-brittle materials: advantages of implicit integration." International Journal for Numerical and Analytical Methods in Geomechanics (2014).
- [3] Vassaux, Maxime, Oliver-Leblond Cécile, Richard, Benjamin and Ragueneau, Frédéric. "Beam-Particle approach to model cracking and energy dissipation in concrete: Identification strategy and Validation". Cement and Concrete Composites (*Under review*).

# Numerical Simulation of Fracturing in Steel Fiber Reinforced Concrete Structures using Interface Solid Elements

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Aiming at the development of a numerical platform for the analyses of structures made of fiber reinforced concrete (FRC), a series of model components are presented, which allows to investigate the effect of various composite parameters (fiber type, fiber content and distribution, concrete strength) at different length scales.

The basic ingredient at the level of single fibers is an analytical model allowing the prediction of the pullout response of a steel fiber, either straight or with hooked end, with or without inclination with respect to the loading direction, taking into account the interfacial slip, the plastic deformation of fiber and the localized damage of concrete [1]. To investigate the bridging effect of an opening macro crack in the FRC composite containing a number of distributed fibers, a representative volume element (RVE) with the crack plane located at the center is considered. The traction-separation relation is obtained via the integration of the pullout responses of all the fibers intercepting the crack, taking the content and distribution of fibers into consideration [2].

For the numerical simulation of fracture processes in FRC at the level of structures, a mesh fragmentation technique based upon *interface solid elements* [3, 4] is used to capture propagating cracks. This approach is characterized by embedding strong discontinuities in finite elements with almost zero thickness which are a priori placed between all bulk elements. The inelastic material laws for the interface solid elements are obtained from crack bridging laws. Selected 2D and 3D numerical examples demonstrate that the fracture paths in plain- and in fiber reinforced concrete structures under tension-dominant loading conditions can be well captured without employing any crack tracking technique (see Fig. 1 and Fig. 2).

## References

[1] Y. Zhan and G. Meschke. Analytical model for the pullout behavior of straight and hooked-end steel fibers. *Journal of Engineering Mechanics (ASCE)*, 140, 04014091 (1-13), 2014.



Figure 1: Simulation of a notched FRC beam under 3-point-bending: (a) Comparison of force vs. crack mouth opening displacement relations for two fiber composites and plain concrete. (b) Deformation (scale = 10) and contour plot of crack opening magnitude.



Figure 2: Simulation of Brokenshire's (torsion) test on plain concrete: (a) Comparison of force vs. crack mouth opening displacement relations. (b) Deformation (scale = 100) and contour plot of crack opening magnitude.

- [2] V.C. Li, Y. Wang and S. Backer. A micromechanical model of tension-softening and bridging toughening of short random fiber reinforced brittle matrix composites. J. Mech. Phys. Solids, 39(5), 607-625, 1991.
- [3] O.L. Manzoli, A.L. Gamino, E.A. Rodrigues and G.K.S. Claro. Modeling of interfaces in two-dimensional problems using solid finite elements with high aspect ratio. *Computers and Structures*, 94-95, 70-82, 2012.
- [4] M. Sánchez, O.L. Manzoli and L.J.N. Guimarães. Modeling 3-D desiccation soil crack networks using a mesh fragmentation technique. *Computers and Geotechnics*, 62, 27-39, 2014.

# Embedded Finite Element Method based on plate-kinematics for reinforced concrete elements

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Recent damage models, based on a continuum description of the media, succeed in representing the main features related to the complex behavior of quasi-brittle materials such as cracking, crack closure effect and permanent strains [1]. Nevertheless, cracking is described in a diffuse way and it is always difficult to quantify the cracking features such as openings and spacing. Post-processing methodologies are needed to estimate the aforementioned quantities [2].

Recently, a new concept of displacement discontinuities embedded into a standard finite element has come up in the continuity of the smeared crack approach. The kinematics of a traditional finite element is enhanced by a displacement jump which represents the crack opening. In our study, the Embedded Finite Element Method (E-FEM) is used [3]. Numerically, the enhancement related to the displacement jump takes places locally in the element. The non-linear behavior is handled by a tension/separation law characterizing the energy dissipation on the discontinuity.

The objective of our study is the development of a kinematic enhanced damage model to represent cracking patterns of reinforced concrete components subjected to seismic loadings. Within the framework of earthquake engineering in which time-consuming nonlinear dynamic analyses have to be carried out to make seismic assessments, the use of reduced kinematic-based elements such as plates and shells is relevant to model reinforced concrete components like slabs or shearwalls. An anisotropic damage model, based on micromechanical assumptions, is used [4]. This model allows accounting, in a natural manner, for particular crack orientations in reinforced concrete membrane elements. A "discrete" damage formulation is considered by introducing *p* couples of microcrack densities and directional tensors, denoted by  $\rho_p N_p$ . Microcrack densities  $\rho_p$  are considered as internal variables and the directional tensors  $N_p$  are constructed as the tensor product of the normal to the crack. The model can represent either mode-I and mode-II cracking mechanisms which can be handled independently.

This model is then enhanced with the strong discontinuity kinematics. Regularization of the Dirac distribution [3] provides a consistent discrete constitutive model expressed in terms of traction/separation law. Some numerical simulations will be shown to illustrate the performances of this model.

## Références

- [1] B. Richard, F. Ragueneau. Continuum damage mechanics based model for quasi brittle materials subjected to cyclic loadings : Formulation, numerical implementation and applications. *Engineering Fracture Mechanics*, 98, 383-406, 2013.
- [2] C. Oliver-Leblond, A. Delaplace, F. Ragueneau, B. Richard. Non-intrusive global/local analysis for the study of fine cracking. *International Journal for Numerical and Analytical Methods in Geomechanics*, 37(8), 973-992, 2013.
- [3] J. Oliver. Modelling strong discontinuities in solid mechanics via strain softening constitutive equations. Part 1 : Fundamentals and Part 2 : Numerical simulation. *International Journal for Numerical Methods in Engineering*, 39(21), 3575-3623, 1996.
- [4] R. Bargellini, D. Halm, A. Dragon. Modelling of quasi-brittle behaviour : a discrete approach coupling anisotropic damage growth and frictional sliding. *European Journal of Mechanics-A/Solids*, 27(4), 564-581, 2008.

# Analysis of Crack Propagation Instabilities in Rock Specimens via Numerical Analysis of Wedge Splitting Tests.

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Wedge Splitting Tests (WST) is a method to perform stable fracture mechanics tests on quasi-fragile materials which provides a relatively simple manner to obtain the specific fracture energy parameter  $G_F^I$  using simple specimens like cubes or cylinders [1]. The stability of the fracture propagation depends on the interaction between of the control parameter chosen (displacement or deformation control), the stiffness of the testing machine, the specimen stiffness and the specimen geometry, as well as the material properties [1].

In this paper, experimental results performed in-house of an unstable WST were simulated by means of the FEM and fracture-based zero-thickness interface elements. Standard elastic continuum elements were used to represent the rock, the steel loading plates and an "equivalent spring" to the testing machine compliance, while interface elements were used for the notch and along the potential crack path. The interface elements representing the notch were equipped with linear elastic constitutive law, with very low elastic stiffness  $K_n$  and  $K_t$ , so that they do not oppose any significant resistance to opening. The constitutive model used for the interface elements along the fracture path was the elastoplastic constitutive model with fracture energy-based evolution laws described in detail in [2].

In order to obtain a specimen geometry suitable for a stable WST without modifying the remaining significant parameters (machine stiffness and control parameter), a number of additional simulations were performed varying the notch length, until a load-COD curve without snap-back was obtained. Finally, a new experimental WST with the modified geometry was performed leading to a stable load-COD curve, which turned out in very good agreement with the predicted one.

- [1] E. Brühwiler & F. Wittmann, The wedge splitting test, a new method of performing stable fracture mechanics tests. Engineering Fracture Mechanics, 35(1), 117– 125, 1990.
- [2] I. Carol, P. Prat, & C. M. López. A normal/shear cracking model. Application to discrete crack analysis. ASCE J. Engrg. Mech., 123(8),765-773,1997.

# Mesoscale modelling of concrete quasi-brittle behaviour

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Fracture of quasibrittle materials as cementitious materials is characterized by the development of a micro-cracked damaged zone ahead of the main crack, called Fracture Process Zone (FPZ). This zone is responsible for the dissipation the elastic energy stored in the structure due to stress redistribution. The characterization of the FPZ constitutes a major challenge for the understanding of concrete mechanical behaviour. In fact, the FPZ has many consequences on the strains localizations, the size effect and the capability of numerical models to reproduce the realistic behaviour of concrete.

The aim of our study is to develop a numerical model without kinematic discontinuity (cracks modelling) which describes damage mechanisms in concrete under mechanical stress ranging from diffuse failure to localisation and final discrete failure. The used damage model is the isotropic damage model developed by Fichant & al. [1]. This later is implemented in the finite elements code Cast3M<sup>(C)</sup>. It is based on the continuum damage mechanics and on the relation between the total stress and the effective stress of the material. This model takes into account the crack closure effects (unilateral effect) and the plasticity. Moreover mesh dependency is partially controlled by an energetic regularization method. In order to have a realistic description of the fracture process and the corresponding effect of concrete heterogeneities, a mesoscale modelling is required [2]. The volume of concrete is defined by two different phases: a matrix consisting of the cement paste or mortar, and aggregates which are randomly placed, from the largest to the smallest, according to the grain size analysis. A diffuse meshing method, with matrix and aggregates properties projected on the shape functions of the finite element mesh was used. This method allows to model particle sizes distributions from ranges to less than mm to severals cm. The mesoscale damage model requires few input parameters (Young modulus, Poisson's ratio, tensile strength, fracture energy) for mortar and aggregates which can be assessed by simple tests. Working on damage and cracks fields from this model is crucial for the FPZ characterization.

Initially, the effect of input parameters on the damage fields and the macroscopic mechanical behaviour was studied. Then, the relevance of modelling all classes of aggregates was investigated. In addition, this study allows to determine the role of different sizes of aggregates on the FPZ development. Finally, according to the equivalent linear elastic fracture mechanics (LEFMeq), the crack growth resistance curves or R-curves were plotted based on the load-displacement curves obtained with the numerical concrete [3]. The results show that the mesoscopic damage modele is able to assess an internal length contrary to non-local damage models in which it is an inherent input parameter.

- [1] S. Fichant. C. La Borderie. G. Pijaudier-Cabot. Isotropic and anisotropic descriptions of damage in concrete structures. *Mechanics of cohesive-frictional materials*, 4, 339-359, 1999.
- [2] C. La Borderie. C. Lawrence. T.D. N'Guyen. A mesoscopic approach for a better understanding of the transition from diffuse damage to localized damage. *Fracture Mechanics on Concrete and Concrete Structures*, FraMCoS'7, 2010.
- [3] S. Morel. R-curve and size effect in quasibrittle fractures: case of notched structures. *International Journal of Solids and Structures*, 44, 4272-4290, 2007.
# A Smooth unloading-reloading approach for modelling quasi-brittle failure

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The nonlinear systems of equations resulting from the finite element modelling of quasibrittle material are most-frequently solved using Newton-based incremental-iterative schemes. However, the poor convergence characteristics and frequent numerical breakdowns of those solutions schemes, when solving problems involving quasi-brittle materials, make the nonlinear finite element analysis of such materials is a truly numerical challenging undertaken. The motivation for the work of this paper is largely because of the aforementioned issues.

An incremental-iterative numerical approach for nonlinear finite element analysis of quasi-brittle materials is presented. The new method, named smooth unloading-reloading (SUR) approach, improves the robustness and convergence properties of a finite element isotropic damage model when applied to facture problems in quasi-brittle materials. The SUR approach uses a target function and an unloading-reloading function to compute an approximate tangent matrix with an incremental-iterative Newton type solution procedure. A predictive-SUR algorithm is also developed for further accelerating the SUR method solution, in which a predictive function is used to for calculating a predictive converged value of a damage evolution parameter.

A number of idealised quasi-brittle fracture examples are used to evaluate the performance of SUR and predictive-SUR approaches. These examples were chosen for their numerical characteristic and are not based upon real experiments or structures. The examples demonstrate that the new SUR method is robust and lead to significant savings in the total number of iterations to achieve convergence, compared to solutions obtained with a reference model with secant unloading-reloading behaviour. Furthermore, results proved that numerical breakdown of the nonlinear solution procedure never arose when SUR approach was carried out. In addition, the numerical results show that the predictive SUR approach is reliable and always resulted in overall iteration numbers equal or below those obtained with the standard SUR approach, in some examples these savings were enormous.

The authors may also explore performance of the SUR methods for continuum damage and strong discontinuity.

- [1] Oliver, J., Cervera, M., Oller, S.and Lublineer, J. Isotropic Damage Models and Smeared Crack Analysis of Concrete: Computer Aided Analysis and design of Concrete Structuers, Proceedings of SCI-C1990, II. International Conference, Austria, 1990. p: 945-957.
- [2] Crisfield, M.A. Non-linear Finite Element Analysis of Solids and Structures. Vol.1.1991, Chichester, UK: John Wiley&Sons.
- [3] Jirásek, M. Damage and Smeared Crack Models. Numerical Modelling of Concrete Cracking, G. Hofstetter and Meschke, G., Editors. 2011, Springer Vienna. p:1-49.

## Numerical simulations to determine the constitutive parameters of the oxide layer in corrosion of reinforced concrete

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When corrosion of the steel reinforcement occurs in concrete structures, cracking of the concrete cover might result, among other effects, from the generation of an expansive oxide layer. In the simulation of this type of cracking, the constitutive parameters of the oxide have been proved to be crucial, as presented in previous conferences. However, there is a lack of experimental information about those parameters, due to the difficulty to perform direct in place measurements.

To obtain information close to the oxide layer that permits to narrow the uncertainties about the oxide, an experimental and numerical study was carried out using as specimens concrete prisms reinforced with a steel tube. Accelerated corrosion tests were carried out and the variation of inner diameter and inner volume of the tube were measured with special instruments, which provided additional information to that obtained in specimens reinforced with a bar, mainly the crack width. Simulations of the tests were carried out, using finite elements with an embedded cohesive crack for the fracture of concrete [1], and expansive joint elements for the oxide [2], and the variation of inner diameter and volume were recorded. For the fracture parameters of concrete, experimental characterization was carried out, while for the parameters of the oxide, a numerical study was necessary. The results in the simulations were compared to those obtained in the experiments, and the influence of the parameters of the oxide was analyzed. Finally, the values for the constitutive parameters that best reproduce the experimental results were determined.

- [1] J.M. Sancho, J. Planas, D.A. Cendón, E. Reyes, J.C. Gálvez. An embedded cohesive crack model for finite element analysis of concrete fracture. *Engineering Fracture Mechanics*, 74, 75–86, 2007.
- [2] B. Sanz, J. Planas and J.M. Sancho. An experimental and numerical study of the pattern of cracking of concrete due to steel reinforcement corrosion. *Engineering Fracture Mechanics*, 114, 26–41, 2013.

# Eigenerosion: extension and applications to heterogeneous media

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In 1921, Griffith introduced the concept of energy release rate in brittle fracture: below a certain amount of energy, no crack extension can occur. A variational formulation of this idea has been proposed by Francfort and Marigo [1] in the late 90's whose a regularization (1) has been recently developed by Schmidt *et al.* [4].

$$F_{\epsilon}(u, \epsilon^*, t) = \underbrace{\int_{\Omega} W(\epsilon(u) - \epsilon^*) dV}_{\text{strain}} - \underbrace{\int_{\Gamma_f} \overline{T} \cdot u dS}_{\text{load}} + \underbrace{G_c \frac{|C_{\epsilon}|}{2\epsilon}}_{\text{crack}}$$
(1)

where  $\overline{T}$  is the applied traction on the part  $\Gamma_f$  of the boundary of the domain  $\Omega$ , W is the elastic strain energy density of the body,  $\varepsilon(u)$  is the strain operator of linear elasticity,  $\varepsilon^*$  is an <u>eigendeformation</u> field [2],  $G_c$  is the critical energy-release rate and  $\epsilon$  is a small length which defines  $C_{\epsilon}$  an  $\epsilon$ -neighbourhood to the crack,  $|\mathcal{D}|$  is the measure of any domain  $\mathcal{D}$ . When  $\epsilon$  tends to zero the standard Griffith's theory is recover.

The minimization of the functional (1) within the finite element framework was firstly realized by Pandolfi and Ortiz [3] in the context of homogeneous elastic media. These authors proposed to model the crack propagation associated to the eigenerosion using a 'killing element' method (deletion of elements verifying the crack propagation criterion). Two physical quantities areinvolved in this process.

• A variation of energy.

$$-\Delta E_K = \underbrace{\frac{1}{2} u^T \Delta S_K u}_{\text{element}} + \underbrace{\frac{1}{2} u^T \Delta S_K S^{-1} \Delta S_K u}_{\text{structure}}$$
(2)

where  $\Delta S_K$  is the stiffness matrix of the *K*-*th* element, *S* is the stiffness matrix of the meshed body where the element *K* is removed and *u* the displacement field before the erosion or equivalently before the assumed crack propagation. For sake of simplicity, [3] have taken into account only the element contribution in the equation (2).

• A crack length increment. For a crack occupying a domain *C*, the size of its extension is evaluated by a neighbourhood process [3]:

$$\delta A(K) = \frac{|(C \cup K)_{\epsilon} \setminus C_{\epsilon}|}{2\epsilon},\tag{3}$$

where  $(C \cup K)_{\epsilon}$  is the current neighbourhood of the crack,  $C_{\epsilon}$  is the previous configuration of the crack-neighbourhood and *K* is the current killed element. When  $\epsilon$  tends to 0,  $\delta A(K)$  approaches the physical increment of the crack.

In the present work, 1/ we investigate a criterion with a both local-global energy, 2/ we propose an extension of the eigenerosion method to fracture dynamics, 3/ we point out and clarify some technical problems related to the crack length increment (new crack-neighbourhood process, crack-boundary interactions, crack-crack interactions) and 4/ we extend the eigenerosion method to <u>heterogeneous media</u>. Each development is validated on benchmark.

The extension to heterogeneous media is based on a local predictor-corrector method. The prediction phase consists in determining the elements that are candidates to the erosion process, i.e. that maximize the ratio  $-\Delta E_K / [\delta A(K) G_c(K)]$  over the structure, where  $G_c(K)$  denotes the critical energy release rate of the *K*-th element depending on the underlying phase. This extension to heterogeneous media is applied to an engineering problem: the study of the crack tortuosity in concrete media. The effect of two distinct parameters is studied: the aggregates volume fraction and the size polydispersity of the aggregates.

Moreover, the extension of the method to the 3D case is actually studied.

- [1] G.A. Francfort and J.-J. Marigo. Revisiting brittle fracture as an energy minimisation problem. Journal of the Mechanics and Physics of Solids, 1319-1342, 1998.
- [2] T. Mura. Mechanics of Defects in Solids, Martinus Nijhorff, 1987.
- [3] A. Pandolfi and M. Ortiz. An eigenerosion approach to brittle fracture. International Journal For Numerical Methods In Engineering, 694-714, 2012.
- [4] B. Schmidt and F. Fraternali and M. Ortiz. Eigenfracture: an eigendeformation approach to variational fracture. Multiscale Modeling and Simulation, 1237-1266, 2009.

# FE analyses of a coupled energetic-statistical size effect in concrete beams under bending

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A size effect in concrete elements causes that both the nominal structural strength  $\sigma_N$  and material brittleness decrease with increasing characteristic specimen dimension *D*. Thus, concrete becomes ductile on a small scale and perfectly brittle on a sufficiently large scale. The reasons of this behaviour are [1]: a) the presence of intense localized regions with a certain volume which precede macro-cracks and contribute to a deterministic size effect and b) a random distribution of material properties contributing to a statistical size effect. The first statistical theory introduced by Weibull [2] postulated that a structure was as strong as its weakest component. The statistical size effect by Weibull ignored a spatial correlation between local material properties and a deterministic size effect. Combining the energetic theory with the Weibull statistical theory, a general energetic-statistical theory was developed by Bazant [1], [3].

Our extensive numerical FE investigations of a coupled energetic-statistical size effect in unnotched concrete beams of a similar geometry under quasi-static three point bending were performed within elasto-plasticity with non-local softening to obtain mesh-independent results [4], [5]. The FE analyses were carried out with four different beam sizes scaled in two dimensions. The deterministic calculations were performed with the uniform distribution of the tensile strength  $f_t$ . In turn, in stochastic calculations, the tensile strength  $f_t$ , fracture energy  $G_F$  and elastic modulus E took the form of spatially correlated random fields described by a Gauss distribution (with the prescribed mean value  $\mu$ , standard deviation  $\sigma$  and correlation length  $l_{cor}$ ). A physical relationship between the cross-correlated distribution of  $f_t$ ,  $G_F$  and Ewas assumed, i.e. the lower  $f_t$ , the lower is E and higher is  $G_F$ . In order to reduce the number of stochastic simulations, the stratified sampling technique was applied [4]. Initially, the FE calculations were performed with the spatially varying  $f_t$  only, the length of the spatial correlation  $l_{cor}$  ranged from 15 mm up to 150 mm and the variation coefficient  $cov=\sigma_{ft}/\mu_{ft}$ (keeping the constant  $\mu_{ft}$ ) varied between 0.15 and 0.20. In the next FE calculations, the simultaneously varying tensile strength  $f_t$ , fracture energy  $G_f$  and modulus of elasticity E. were assumed. The strong or weak cross-correlation between  $f_t$ ,  $G_f$  and E was taken into account. The numerical outcomes showed a strong coupled energetic-statistical size effect in concrete beams [4]. The influence of the correlation length  $l_{cor}$  was found to be strong with respect to the variation coefficient of the stochastic nominal strength. The higher  $l_{cor}$ , the higher was the coefficient of variation. A similar influence had the increasing standard deviation  $\sigma_{ft}$ . The simultaneously spatially varied parameters  $f_t$ ,  $G_f$  and E slightly affected the mean stochastic nominal strength.

#### References

[1] Bazant, Z., Planas, J. Fracture and size effect in concrete and other quasi-brittle materials. CRC Press LLC, 1989.

[2] Weibull, W. A statistical theory of the strength of materials. *Journal of Applied Mechanics* 18(9), 293-297, 1951.

[3] Bazant, Z.P., Novak, D. Proposal for standard test of modulus of rupture of concrete with its size dependence. *ACI Materials Journal* 98(1), 79-87, 2001.

[4] Syroka, E., Tejchman, J., Mróz, Z. FE calculations of a deterministic and statistical size effect in concrete under bending within stochastic elasto-plasticity and non-local softening. *Engineering Structures* 48, 205-219, 2013.

[5] Korol, E. Tejchman, J., Mróz, Z. FE analysis of size effects in reinforced concrete beams without shear reinforcement based on stochastic elasto-plasticity with non-local softening. *Finite Elements in Analysis and Design* 88, 25-41, 2014.

## Experimental and numerical investigations of the correlations involved during failure in quasi-brittle materials

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Fracture of quasi-brittle materials such as concrete or rocks is characterized by a macro crack surrounded by a damage zone. At the tip of the macro crack and ahead lies the so-called Fracture Process Zone (FPZ) which is a region of the material undergoing distributed damage. The size of the FPZ in these heterogeneous materials is large enough to influence the mechanical behaviour of the structure significantly. It does not depend on the structural size, but it is rather controlled by the local heterogeneities in the material as well as by the geometry of the specimen and the stress conditions. Therefore, size effect, understood here as the dependence of the dimensionless nominal strength of a structure on its size, is observed.

Experimentally, this damage zone may be characterized with the help of several direct and indirect techniques. The localization of acoustic events that can be detected during crack propagation is one well established technique from which the FPZ can be visualized and characterized. The acoustic events generated during micro-cracking are recorded and post-processed in order to localize them with the help of time-of-flight algorithms. Hence, this technique provides information on the entire crack propagation process composed of distributed micro cracking and further coalescence into a macro crack.

As far as modeling is concerned, continuum based approaches and discrete or mesoscale models are available. The first one involves a characteristic length which controls the size of the FPZ. In recent models, however, it has been pointed out that this internal length is not constant during the fracture process and also that it is influenced by boundaries, which could be expected since experimental works on fracture in concrete underline the influence of boundaries on the fracture energy. The second approach relies on a mesoscale description of the material and on an explicit description of the heterogeneities in the material. As opposed to the continuum approach, mesoscale models do not introduce any characteristic length. At the scale of a lattice element or a discrete element, softening is introduced as a local property.

Grassl and co-workers [1] demonstrated that mesoscale modeling was very efficient at describing not only size effect on the peak load, but also the entire load deflection response of bending beams. Four geometrically similar sizes and three different notch lengths were considered. The experimental data obtained by Grégoire et al. [2] could be quite accurately described, once the model parameters at the mesoscale level had been calibrated for one notch length. In addition, the authors used this model for studying the incremental distribution of the dissipated energy densities, and they were able to track the evolution of the fracture process zone in the structure, depending on the size of the beams and on the boundary conditions.

In addition, Grégoire and co-workers [3] demonstrated that this mesoscale approach is also capable to capture the local failure process realistically. Three point bending experiments coupled with acoustic emission analyses provided global responses of the same bending beams and local data in the form of the distribution of the acoustic events and its evolution in the course of fracture. The experimental data obtained by Grégoire et al. [3], in term of energy dissipation maps and histograms of the distances between damage events, could be quite accurately described with the same set of model parameters. Particularly, the agreement between the distributions of the relative distances between damage events show that the mesoscale model depicts the fracture process zone and its evolution during failure in a very consistent way compared to acoustic emission data. Unfortunately, and contrary to the case of direct tension, these histograms cannot be interpreted easily because the effect of the strain gradient in bending beams cannot be easily separated from the interaction between damage events that may develop in the course of fracture.

The purpose of this paper is to provide a further insight in the description of failure with the help of statistical analyses of damage. The statistical analysis relies on the implementation of Ripley's functions [4], which have been developed in order to exhibit patterns in image analyses. First, it is shown how Ripley's functions may be used in the context of damage mechanics to extract correlation length between damage events. Then, comparisons are presented showing the evolution of extracted correlation lengths during mesoscale numerical simulations and experimental three point bending tests where damage events are localized by acoustic emission techniques . Finally, numerical investigations of correlation length evolutions upon failure are presented for both direct tension and three point bending specimens. Results show that the computed correlation length is not constant during failure and significant differences may be observed depending on the type of loading applied to the same specimen.

- [1] P. Grassl, D. Grégoire, L. B. Rojas-Solano, and G. Pijaudier-Cabot. Meso-scale modelling of the size effect on the fracture process zone of concrete. *International Journal of Solids and Structures*, 49(13):1818–1827, 2012.
- [2] D. Grégoire, L. B. Rojas-Solano, and G. Pijaudier-Cabot. Failure and size effect for notched and unnotched concrete beams. *Int. J. Numerical and Analytical Methods in Geomechanics*, 37(10):1434–1452, 2013.
- [3] D. Grégoire, L. B. Rojas-Solano, V. Lefort, P. Grassl, J. Saliba, J.-P. Regoin, A. Loukili, and G. Pijaudier-Cabot. Mesoscale analysis of failure in quasi-brittle materials: comparison between lattice model and acoustic emission data. *International Journal of Numerical and Analytical Methods in Geomechanics*, page submitted, 2014.
- [4] B. D. Ripley. Modelling Spatial Patterns. *Journal of the Royal Statistical Society*. *Series B (Methodological)*, 39(2):172–212, 1977.

## General sequentially linear method for saw-teeth constitutive relations

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The modelling of fracture in concrete structural components has been under investigation for several decades. Many efficient and robust material models are available today. Usually, a solution is obtained by applying load incrementally in a number of small steps in which an iterative algorithm is employed. The robustness of the chosen iterative scheme is of importance. Problems with the convergence of iterative methods in the case of bifurcations, snap-backs, sudden changes in constitutive law, etc., have led to the development of non-iterative methods.

Non-iterative methods can be applied in connection with damage-based saw-tooth constitutive laws composed of linear segments (which when extended pass through the origin of the stress-strain space) followed by immediate jumps to lower stiffness. The classical load-unload (L-U) solution method is usually used. It was developed for lattices, but can be applied also in connection with finite element method [4].

The L-U solution scheme proceeds simply by loading the linear system until one (critical) element reaches its strength. The critical element is damaged or completely removed and a new stiffness matrix is evaluated for the next step. The whole load is released then the system with reduced stiffness is loaded again until another rupture occurs.

Another method applicable to the saw-tooth type of constitutive law is the forcerelease (*F-R*) method developed to simulate brittle fracture in lattice models [3] and to improve the sequentially linear concept for non-proportional loading paths [2]. The *F-R* method resembles dynamic process of redistributing released stresses from damaged elements while the external load is kept unchanged. During the redistribution, the system evolves from static equilibrium state through disequilibrium states until the static equilibrium is reached again. In contrast with the *L-U* method, the *F-R* algorithm does not unload to the origin after every rupture, but keeps the whole previously applied load acting. It starts from an equilibrium state (initially, the load-free structure), and after rupture in critical element occurs, sequentially linear process of redistributing released stresses is performed.Within the redistribution, other elements may break.

It has been shown that L-U and F-R methods yield different results [3]. It is difficult to judge which method is better, especially for the proportional loading path. Both of the methods are rough approximations of a complex dynamic process. Therefore, the consideration of time scales allows some comparison between the methods.

The contribution shows that the *F*-*R* method is suitable for processes where the redistribution is much faster than the reaction speed of the external load, whereas the *L*-*U* method is good choice for systems with long redistribution times compared to the reaction time of the external load. Moreover, a *general* method that connects the *L*-*U* and *F*-*R* algorithm is developed. This method is essentially nothing else but the *F*-*R* method extended to allow the modification of the external load during redistribution. Both *L*-*U* and *F*-*R* methods can be viewed as special cases of the *general* method.

The *general* algorithm can be used with indirect control of the simulation and also in connection with non-proportional loading paths. More details and examples can be found in [1].

## Acknowledgement

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- [1] Eliáš, J., Generalization of load-unload and force-release sequentially linear methods, *International Journal of Damage Mechanics*, 24(2), 279–293, 2015.
- [2] Eliáš, J., Frantík, P., Vořechovský M., Improved sequentially linear solution procedure, *Engineering Fracture Mechanics*, 77(12), 2263–2276, 2010.
- [3] Liu, J. X., El Sayed, T., On the load-unload (L-U) and force-release (F-R) algorithms for simulating brittle fracture processes via lattice models, *International Journal of Damage Mechanics*, 1(7), 960–988, 2012.
- [4] Rots, J. G., Invernizzi, S., Regularized sequentially linear saw-tooth softening model, *International Journal for Numerical and Analytical Methods in Geomechanics*, 28(7-8), 821–856, 2004.

- chapter 12 -



MS16: Theory of fracture, crack propagation criteria, and crack tracking algorithms

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## From micro- to macroscale fracture properties of highly heterogeneous interfaces: towards a perturbation-based numerical method?

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crack propagation regimes

 Macroscopic effective fractur properties

FIGURE 1 – Failure regimes and effective fracture properties of fracture planes with a heterogeneous distribution of local toughness.  $\Delta$  denotes the relative amplitude of toughness contrast and *k* the number of the heterogeneities along the crack front.

Effects of small scale heterogeneities on the macroscopic fracture properties of materials are various, going from crack bridging, deflection, screening... to crack front pinning. Quantifying these effects is a complex problem in mechanics involving several scales, namely the heterogeneity, crack and body sizes. Here, among these effects we focus on crack front pinning. The basic mechanism is that the crack advance is slowed down or even pinned at places of the crack front facing some stronger material. We make the further assumptions of (i) brittle fracture, (ii) tensile, mode I loading and of (iii) planar crack propagation. They apply in particular to cracks propagating along a textured interface, as one can encounter in multifunction double glazing or in electronic chips. The situations investigated here are also representative of crack pinning phenomena encountered in many materials at their microstructure scale. This study is thus a first step toward the description of more complex situations. The question we aim to deal with is as follows : How can we predict the fracture properties at the large scale from the knowledge of the toughness map at the small scale ? This requires the definition of some macroscopic toughness, and a method to calculate them. Here, we illustrate the difficulty of the task on some model toughness map, chosen to be as simple as possible, but complex enough, to catch some possible effects of large crack front deformations.

More precisely, we consider the propagation of a circular crack in a particular heterogeneous toughness map, of radial symmetry, for which the toughness is invariant in the propagation direction. This map allows us to study the combined effects of toughness contrast level and heterogeneity size. At the microscale, the crack advance is driven quasistatically by Irwin's or equivalently Griffith's local thresholds,  $\kappa_c(M)$  and  $\gamma_c(M)$ . At the macroscale, some definition of a macroscopic stress intensity factor K and energy release rate G are introduced leading to some Irwin's and Griffith's criterions involving some effective macroscopic toughness  $K_c$  and fracture energy  $G_c$ . To solve the problem [1], we use the finite perturbation method, proposed by Rice (1989) and further developed numerically by Bower and Ortiz (1990) and Lazarus (2003, 2011). It is based on numerical multiple iterations of first order perturbation approaches. Sole the crack front has to be meshed making the method efficient and sensitive enough to solve our problem involving the interaction of the crack front with local heterogeneities.

Different propagation regimes are found depending jointly on the contrast level and the heterogeneity size (fig. 1(a)) : the crack front may reach, or not, a shape for which the local stress intensity factor equals the local toughness at each point of the front [1]. For small enough contrasts, both macroscopic Irwin and Griffith's thresholds are found to be equivalent and the macroscopic quantities  $K_c$  and  $G_c$  are equal to the corresponding mean value,  $\overline{\kappa_c}$  and  $\overline{\gamma_c}$ , of the local maps  $\kappa_c$  and  $\gamma_c$ . For larger contrasts, both quantities decrease with the toughness contrast and the heterogeneity size, from their average down to their minimum local values (fig. 1(b)).

This study is a first step toward homogeneisation in brittle fracture for highly heterogeneous material, taking into account the deformation of the crack front induced by the heterogeneities. We envisage now to apply it to more complex toughness maps and to compare the results with some experiments analoguous, albeit more complex, to those of [2, 3]. Further developments of the current method include its extension to dynamic propagation and out-of-plane excursions of the crack.

### Références

- M. Vasoya, V. Lazarus, and L. Ponson. Crack front fingering during planar crack propagation in highly heterogeneous toughness field. *Procedia Materials Science*, 3:2142–2147, 2014.
- [2] S. Xia, L. Ponson, G. Ravichandran, and K. Bhattacharya. Toughening and asymmetry in peeling of heterogeneous adhesives. *Phys. Rev. Lett.*, 108 :196101, 2012.
- [3] S. Patinet, L. Alzate, E. Barthel, D. Dalmas, D. Vandembroucq, and V. Lazarus. Finite size effects on crack front pinning at heterogeneous planar interfaces : experimental, finite elements and perturbation approaches. *Journal of the Mechanics and Physics* of Solids, 61(2):311–324, 2013.

# Effect of Non-Uniform Stress Field on Bi-dimensional Cohesive Crack Initiation and Propagation

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Griffith's theory of fracture [1] based on the concept of critical energy release rate  $G_c$  remains the most used in fracture mechanics thanks to its simplicity in terms of material behavior. However, this theory contains many major drawbacks. In particular, the stress singularity is present at crack tips and it is impossible to initiate crack from a sound body. Basing on ideas of Dugdale [2] and Barenblatt [3], cohesive-zones models have been proposed since 1960 to extend the drawbacks of Griffith's theory. According to the models, a cohesive zone in which the crack lips are subjected by cohesive force locates between the cracked part and the sound part of the body. Cohesive-zone models represent a widely used alternative method to analyze crack growth.

This paper is interested in the effect of non-uniform stress field on cohesive crack initiation and propagation in bi-dimensional elastic infinite structure through complex analysis ([4], [5]). The loading is monotonic and  $\Delta$  denotes the loading parameter. In the first part, the Dugdale's cohesive law is investigated. Using this cohesive law, the cohesive stress is equal to  $\sigma_c$  along the crack while the loading parameter is smaller than a critical value  $\Delta_c$ . The crack is assumed to be always on the symmetrical plan of structure and in mode I. It should be demonstrated that the crack length is proportional to  $\sqrt{\Delta}$ . This result leads us to introduce a characteristic length  $\ell$  which relates to the non-uniformity of stress field. The crack opening can be also calculated as a regular function of coordinate and of loading parameter  $\Delta$ . This function is maximal at the center of crack and its derivative is equal to 0 at crack tips. The analytical results fit perfectly with numerical simulation which is realized with finite element software Code\_Aster (see Fig. 1). At the critical loading  $\Delta_c$ , the stress-free zone appears in the center of crack. It is shown that the crack lengths-loading parameter curves contain the snap-backs and the crack lengths evolution is discontinuous at  $\Delta_c$ . The scale effect will be discussed in this part of the paper.





In the second part, the paper focus on a more general problem using Barenblatt's linear cohesive law. The dependence of cohesive force on displacement jump is considered. The non-singularity condition of stress field and the complex analysis lead us to the resolution of an integro-differential equation concerning the dimensionless crack opening. Developing the latter as a sum of Chebyshev's polynomials of the dimensionless coordinate, the crack evolution can be investigated approximately. It is demonstrated that the dimensionless crack length. The results of Dugdale's cohesive crack can also be verified by the Chebyshev's polynomials method.

The work is part of studies on the understanding of the crack initiation and propagation phenomena in plastic medium for EDF's industrial problems.

## Références

- [1] A. Griffith, *The phenomena of rupture and flow in solids*, Philos. Trans. Roy. Soc. London, 1920
- [2] D.S. Dugdale, Yielding of steel sheets containing slits, J. Mech. Phys. Solids, 1960
- [3] G.I. Barenblatt, *The methematical theory of equilibrium cracks in brittle fracture*, Adv. Appl. Mech., 1962
- [4] Muskhelishvili, *Some basic problems of mathematical theory of elasticity*, P. Noo-rhoff Ltd, Groningen, 1963
- [5] G.V. Kolosov, On some application of the theory of functions of a complex variable to plane problem of mathematical elasticity theory, Mattison, Yuriev, 1909

## **3D crack propagation with X-FEM cohesive elements**

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In case of brittle failure of metallic components or reinforced concrete structures, trajectories are often unknown in case of mixed mode loading. The X-FEM cohesive model presented in this talk will address the issues of crack bifurcation and crack advance.

A procedure in four steps is adopted: computation of the equilibrium state in the presence of cohesive forces with a given potential crack surface, detection of the updated crack front on the surface from the computed cohesive state, determination of bifurcation angles along the front, and update of the potential crack surface accordingly.

The cohesive model that is used [1] allows initial perfect adherence. It relies on the use of an XFEM-suited reduced space of Lagrange multipliers [2, 3], on the use of a mortar formulation to write the cohesive law from quantities defined over this space in an appropriate manner, and finally on a lumping strategy leading to block-wise diagonal operators.

The originality of the approach lies in the a posteriori computation of the crack advance speed that is naturally embedded in the cohesive model, while in most of the literature it is determined beforehand based on the stress state ahead of the front. A first rough updated crack front is computed from the internal variables of the cohesive law. This rough crack front is then converted into a smooth crack advance speed, itself converted back into a new smooth crack front location with the help of the level-set formalism.

The crack bifurcation angle is determined along the front, based on a criterion using the equivalent stress intensity factors. A new way of computing these equivalent stress intensity factors is suggested, from the cohesive fields exclusively. Then, the potential crack surface is updated accordingly based on an explicit level-set update algorithm [4].

Finally, several numerical tests have been carried out in mixed mode I and II to reproduce 3D non planar crack paths and showed good accordance with previous results from the literature.

- [1] E. Lorentz. A mixed interface finite element for cohesive zone models. Computer methods in applied mechanics and engineering, 198:317-320, 2008.
- [2] N. Moës, E. Béchet, and M. Tourbier. Imposing dirichlet boundary conditions in the extended finite element method. Int. J. Num. Meth. Engng, 67:1641-1669, 2006.
- [3] S. Géniaut, P. Massin, and N. Moës. A stable 3d contact formulation for cracks using x-fem. Revue Européenne de Mécanique Numérique, 75:259-276, 2007.
- [4] D. Colombo. An implicit geometrical approach to level-set update for 3d non-planar x-fem crack propagation. Comp. Meth. Appl. Mech. Engng., 240:39-50, 2012

## Finite-element modeling and analyses for quasicrystals with cracks

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The quasicrystals (QC), short for quasiperiodic crystals, have long-range orientational order but without translational symmetry in particular direction. With such special atom arrangement the quasicrystal is a new material class besides crystals and non-crystals (amorphous solids). Crystallographically, quasicrystals can be categorised into some subclasses (1D, 2D or 3D) which depends on how many quasiperiodic directions there are, e.g. one dimensional quasicrystals have two periodic atom arrangement directions and one quasiperiodic direction [1]. Since the first discovery of quasicrystals in a man made Al-Mn alloy about thirty years ago, people made great effort to research this kind of outstanding material. The investigation of physical properties of quasicrystals, such as the fracture behavior, electronic properties, hardness etc., is essential extending their fields of application. Today, QC are known to exhibit e.g. a very good wear resistance, low friction coefficients and to possess a very low porosity.

This work focuses on macroscopic cracks in 1D quasicrystalline plates, where the material is governed by constitutive equations under the consideration of quasicrystal linear elasticity theory. Analytically, the explicit solutions for the coupled phonon-phason fields of quasicrystals are derived applying a generalized Stroh formalism [2]. The numerical model and simulation for arbitrary cracks are established in a FE environment, where the constitutive behavior of coupled phonon and phason fields is included in special user elements. Different numerical techniques like a displacement interpretation method or modified crack closure integral are applied in a generalized form to calculate stress intensity factors and energy release rates. The validity of conventional fracture criteria and the influence of coupled fields on the mechanical and fracture behavior of quasicrystals are finally investigated.

- [1] T.Y. Fan and Y.W. Mai. Elasticity theory, fracture mechanics, and some relevant thermal properties of quasi-crystalline materials. *Appl. Mech. Rev.* 57(5), (2004), 325–343.
- [2] Y. Gao, A. Ricoeur, L.L. Zhang, L.Z. Yang. Crack solutions and weight functions for plane problems in three-dimensional quasicrystals. *Arch. Appl. Mech.* 84(8), (2014), 1103–1115.

## Assessment of Size-Dependent Fatigue Failure Modes Using a Cyclic Cohesive Zone Model

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A cyclic cohesive zone model is applied to several problems of fatigue with different failure modes.

In contrast to comparable cohesive models, a local endurance limit is embedded that decreases with increasing damage state. Each of the cohesive parameters has a distinct physical meaning as described in [1]. In preceding studies, fatigue crack growth of a crack under  $K_{\rm I}$ -controlled cycling loading as well as failure of smooth specimens under homogeneous stress-controlled cyclic loading have been investigated [1, 2]. The present contribution tries to close the gap between the toughness-based and the strength-based fatigue criteria. The underlying hypothesis of this concept is, that the intrinsic fatigue mechanism of the material is sufficiently described by the cyclic cohesive zone model. The particular fatigue failure mode depends on the geometry of the specific structure and the loading conditions.

A finite element model of a modified DCB specimen is used to investigate the dependency of size on fatigue failure (compare [3]). The fatigue behaviour of both aforementioned studies is recovered. A fatigue criterion is developed which allows to predict the failure mode of the structure for given size, cohesive parameters and cyclic load parameters. Cyclic failure maps are presented. They allow to assess the cyclic material behaviour by separating endurability, fatigue and static failure.

- [1] S. Roth, G. Hütter, M. Kuna. Simulation of fatigue crack growth with a cyclic cohesive zone model. *International Journal of Fracture*, 188, 23-45, 2014.
- [2] S. Roth, M. Kuna. Fatigue Modelling with a Cyclic Cohesive Zone Approach. *Procedia Materials Science, 20th European Conference on Fracture,* 3, 325-330, 2014.
- [3] B. Wang, T. Siegmund. A computational analysis of size effects in fatigue failure. *Modelling and Simulation in Materials Science and Engineering*, 14, 775-787, 2006.

## Crack path predictions and experiments in structures with interfaces

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A cornerstone of the modern lightweight technology are engineering structures consisting of composite materials combining and enhancing the features of the employed materials. For a safe and reliable application of such composites, amongst others, special attention must be directed to their fracture behavior and the growth of cracks. To obtain a general understanding of the evolution of crack paths in composites, plane specimens with perfect and imperfect material interfaces are investigated. The paired materials exhibit substantial differences in their properties e.g. Young's modulus or fracture toughness, in general providing a mixed-mode loading leading to curved crack paths.

The numerical model for the predication of crack paths is based on a smart remeshing algorithm and an incremental crack extension. For the crack tip loading analysis pathindependent integrals are used such as the interaction integral ( $I_k$ -integral) [1],  $J_k$ -, Mand L-integrals [2]. Large integration contours provide accurate crack loading quantities essentially exploiting reliable numerical data. Considering curved cracks for the sake of path-independence special attention must be directed to the calculation of crack face integrals [3, 4]. Applying large contours, an integration along the material interface is necessary to distinguish between the material forces acting on defects in the body i.e. the crack tip and the interface. Finally, the local anisotropies in elastic constants or the fracture toughness are crucial parameters for an accurate prediction of crack paths [5, 6]. Numerical crack paths in different types of specimens with interfaces are presented and compared with experiments.

- [1] Chen H.K. and Shield R.T., Conservation laws in elasticity of the J-integral type. *Journal of Applied Mathematics and Physics*, 28:1–22, 1977.
- [2] Budiansky B. and Rice J.R., Conservation laws and energy-release rates. *Journal of Applied Mechanics*, 40(1):201–203, 1973.
- [3] Judt P.O. and Ricoeur A., Accurate loading analyses of curved cracks under mixedmode conditions applying the J-integral. *International Journal of Fracture*, 182:53– 66, 2013.

- [4] Judt P.O. and Ricoeur A., Consistent application of path-independent interaction integrals to arbitrary curved crack faces. *Archive of Applied Mechanics*, 85:13–27, 2015.
- [5] Judt P.O. and Ricoeur A. and Linek G. Crack paths at multiple-crack systems in anisotropic structures: simulation and experiment In *Proceedia Materials Science*, *Proceedings of the ECF20, Trondheim, Norway*, 3:2122–2127, 2014.
- [6] Judt P.O. and Ricoeur A. and Linek G., Crack path prediction in rolled aluminum plates with fracture toughness orthotropy and experimental validation. *submitted to Engineering Fracture Mechanics*, 2015

## Plasticity-induced crack closure during cyclic propagation: numerical prediction of crack shape evolution

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The design of many critical structural components needs a careful description of the propagation characteristics of fatigue cracks. The propagation of long and short cracks has been extensively studied, and the role of crack closure has been widely confirmed.

Crack front curvature is commonly obtained in most fatigue experiments. Nevertheless, classical linear elastic fracture mechanics generally considers bidimensional crack geometries corresponding to straight crack fronts, and only few studies consider 3D effects [1].

The objective of this study is then to provide a 3D numerical predictive tool of the evolution of the crack front curvature during the propagation of a fatigue crack taking into account plasticity-induced crack closure.

Tests carried out in CT-50 specimen of 304L austenitic stainless steel have given the stabilized crack front shape for a long crack. In order to avoid any influence of the loading history, constant stress intensity factor (SIF) ranges  $\Delta K$  have been applied.

A 3D numerical tool, using the ABAQUS<sup>®</sup> code and the programming language PYTHON has been developed. A frictionless contact placed on the crack plane allows determining the opening load. Two parallel calculations are done (elastic and plastic), in order to measure on each node of the current crack front the value of the local effective SIF range  $\Delta K_{eff}^{\ell}$ , which has been assumed to be the driving force for the whole propagation. The resulting advance of each node then gives the next crack front, after different mathematical interpolations. A remeshing procedure is then developed, and numerous steps are done until stabilization of the predicted crack shape.

Comparison of experiments and prediction shows globally a good agreement. Further investigation still in progress seems to demonstrate that the introduction of a constraint factor in the tool to consider more accurately the edge effects brings improvement.

#### References

[1] Vor K, Gardin C, Sarrazin-Baudoux C, Petit J. Wake length and loading history effects on crack closure of through-thickness long and short cracks in 304L: Part II- 3D numerical simulation. Eng. Fract. Mech. 2013; 99:306-323.

# Three-dimensional crack nucleation, growth and coalescence using the Thick Level Set approach to fracture

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The thick level set (TLS) model is a unified theoretical model able to model nucleation, growth and coalescence of cracks. It is based on a non-local model for damage. Its originality with respect to existing other non-local damage model (integral, second order, phase-field, ...) is twofold.

First, the boundary of the fully damage area (crack) is explicitly given by a level set. The introduction of a discontinuous kinematic is thus made easy in particular with the extended finite element method (X-FEM). The crack placement adjusts exactly to damage growth. This avoids drawbacks of ad hoc crack placement in damage zone, that is: convergence issue of the global solve if the crack is placed too late and abrupt loss or energy if the crack is placed too quickly. In the latter case, a remedy is to introduce a cohesive crack but this requires yet another model on top of the damage model.

The second originality of the TLS is that non-local damage is restricted to a narrow band around the crack. The extra cost to deal with non-locality is thus small.

Both originalities stem from the fact that the non-locality of damage is formulated by an Eikonal constraint and not a Laplacian constraint.

*Three-dimensional examples will demonstrate the capability of the TLS to model crack initiation, growth and coalescence.* 

- [1] Moës, N., Stolz, C., Bernard, P.-E., & Chevaugeon, N. (2011). A level set based model for damage growth : the thick level set approach. *International Journal For Numerical Methods in Engineering*, 86, 358–380.
- [2] Bernard, P.-E., Moës, N., & Chevaugeon, N. (2012). Damage growth modeling using the Thick Level Set (TLS) approach: Efficient discretization for quasistatic loadings. *Computer Methods in Applied Mechanics and Engineering*, 233-236, 11–27.
- [3] Stolz, C., & Moës, N. (2012). A new model of damage : a moving thick layer approach. *International Journal of Fracture*, 174(1), 49–60.

# Modelling of Quasi-Brittle Fracture in Large Scale Coarse Mesh Finite Element Models using the FEMME (Finite Element Microstructure MEshfre) Multiscale Model

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In quasi-brittle materials, the use of a coarse mesh in a cohesive Finite Element Model over estimates the fracture energy released via damage of the component, and consequently a solution far from its real behaviour may be obtained. We have developed the Finite Element Microstructure MEshfree (FEMME) model [1] that introduces the microstructure as a local enrichment in the damaged zones of the largescale FE mesh. The model is an evolution of the general CAFE model [2] and its composed by 3 layers, the FE model with a coarse mesh that deals with the large scale model, a Microstructural Adaptive Meshfree (MAM) model that uses the isogeometry of the microstructural features (simplified as ellipsoids with random orientation and size) to construct its local maximum entropy exponential shape functions [3], and a Cellular Automata model that reproduces the different fracture properties of the microstructure, that can be extracted directly from experimental data or a lower scale model, and computes from the results of the MAM model the strain of each cell. With this methodology the strain of every cell can be evaluated against its critical value, and the damage of the microstructure can be computed accordingly. Once damage occurs, the mechanical properties of the different layers are recomputed with the Hill theorem [4].

This allows us to insert locally in the large-scale FE model with a coarse mesh the fracture processes of the microstructure, reproducing the Fracture Process Zone explicitly and releasing the spurious fracture energy of the FEM with a coarse mesh.

### References

[1] L. Saucedo-Mora and T.J. Marrow. FEMME: a multi-scale Finite Element Microstructure Meshfree fracture model for quasi-brittle materials with complex microstructures. Submitted to Engineering fracture mechanics.

- [2] A. Shterenlikht and I.C. Howard, The CAFE model of fracture-application to a TMCR steel. Fatigue and Fracture of Engineering Materials and Structures, 29 (9-10), 770-787, 2006
- [3] M. Arroyo and M. Ortiz. Local maximum-entropy approximation schemes: a seamless bridge between finite elements and meshfree methods. International Journal for Numerical Methods, 65, 2167-2202, 2006.
- [4] R. Hill. On the micro-macro transition in constitutive analysis of elastoplastic response at finite strain. Math. Proc. Camb. Phil. Soc, 98, 579-585, 1985.

# Crack-surface based path-following control for phase-field modeling of brittle fracture

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In our research we work toward the development of an accurate and versatile numerical tool for modeling problems in brittle fracture mechanics. For many applications – such as hydraulic fracturing – multiple crack nucleation, branching, merging and propagation are common phenomena. Complex crack patterns appear as a consequence of *e.g.* the presence of multiple cracks and the anisotropy and heterogeneity of the environmental conditions. Using discrete fracture models it can be difficult to capture such topologically complex crack patterns, which has led to the development of phase-field models [1, 2]. In phase-field models the crack surface is regularized by a smeared damage function, which avoids the need for the explicit tracking of fracture surfaces.

In this contribution we consider the quasi-static evolution of brittle fractures in an elastic solid. Path-following control is needed to adequately track the complete equilibrium path [3]. The general strategy in path-following techniques is to supplement the system of non-linear equations with a path-following constraint [4,5]. With the iterative incrementation of the path parameter, a discrete parametrization of the equilibrium path in terms of the path-following parameter can then be obtained. For a phase-field model, the available explicit representation of the crack surface area can be chosen as this control parameter. This choice ensures energy dissipation during the fracture process, which is consistent with the irreversibility of the fracture process.

We investigated three algorithms to solve the non-linear boundary value problem: a monolithic, a staggered and a semi-staggered scheme (see Figure 1). In the monolithic scheme, a Newton-Raphson procedure for the non-linear system of equations (the elasticity equation and the phase-field equation) coupled with a load-control equation is performed. The simulation is controlled with the crack surface area as the control parameter. In the staggered scheme, we solve the phase-field equation and then solve the elasticity equation with an updated phase field (no control procedure is used). Our novel contribution in this work is the development of a semi-staggered path-following scheme. We first solve the displacement sub-problem with a path-following constraint, followed by the solution of the phase-field equation.



Figure 1: Single edge notched tension test [1]

The monolithic scheme computes exact equilibrium points at larger computational cost with the additional advantage of step size insensitivity. The staggered scheme is easy to implement and gives approximate solutions at significantly lower computational cost. However, it is to be noted that the equilibrium path is not exact and highly sensitive to the step size. The semi-staggered path-following scheme proposed herein is highly robust and computationally significantly cheaper than the monolithic scheme. It is less dependent on the step size compared to the staggered scheme. The semi-staggered path-following scheme is shown to yield results that are in excellent agreement with benchmark problems in literature [1].

- C. Miehe *et. al.*, A phase field model for rate-independent crack propagation: Robust algorithmic implementation based on operator splits, *Comput. Methods Appl. Mech. Engng.* 2010, 199: 2765-2778
- [2] M.J. Borden et. al., A phase-field description of dynamic brittle fracture, Comput. Methods Appl. Mech. Engng. 2012, 217-220: 77-95
- [3] R. de Borst *et. al.*, Non-linear Finite Element Analysis of Solids and Structures, *John Wiley & Sons Ltd.*, 2012
- [4] M.A. Gutiérrez, *Energy release rate control for numerical simulations of failure in quasi-brittle solids*, Commun. Numer. Meth. Engng. 2004, 20: 19-29
- [5] C.V. Verhoosel *et. al. A dissipation-based arc-length method for robust simulation of brittle and ductile fracture*, Int. J. Numer. Meth. Engng. 2009; 77:1290-1321

# Fast marching method for three-dimensional crack propagation

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In the recent past years, numerical modeling of fracture problems has attracted growing interest. In the context of the X-FEM method [1], the use of two level sets allows a localization and representation of the crack and its front to be achieved regardless of the mesh. The crack surface is characterized as the zero of one of these level set functions while the crack front is localized as the iso-zero of both level sets.

The simulation of crack propagation is based on updating both level sets. The main difficulty associated with the method involves rebuilding them while keeping the important property of being a signed distance function, at each propagation step.

Several techniques exist in the literature to deal with the evolution of the level set functions: the resolution of Hamilton-Jacobi equations [4], an implicit geometrical approach to level sets update [5] and a mesh evolution tracking method which uses an auxiliary discretization of the crack surface [6] independent of the structural mesh in which it is embedded. We propose here a method based on fast marching [2,3] that consists in propagating the information «distance» from the points of lowest distance with respect to the iso-zero to the furthest regions on a neighbor to neighbor basis.

The method is applied to 2D and 3D meshes and valid for all types of elements (triangles, quadrangles, tetrahedrons, hexahedrons, pentahedrons) of a standard finite element library, which constitutes an interesting contribution for a generalized industrial free software in mechanical engineering such as Code\_Aster (http: //www.code-aster.org) developed by EDF R&D. Moreover it is compared with the other techniques cited above in terms of robustness, numerical efficiency and quality of the results and shows very nice properties.

- N. MOES, J. DOLBOW, and T. BELYTSCHKO. A finite element method for crack growth without remeshing. International Journal for Numerical Methods in Engineering, 46 :135–150, 1999.
- [2] J.A. SETHIAN. A fast marching level set method for monotonically advancing. Proc.Natl.Acad.Sci.USA, 93 :1591–1595, 1996.
- [3] R. KIMMEL and J. SETHIAN. Computing geodesics path on manifolds. Proceedings of National Academy of Sciences, 95 :8431–8435, 1998.

- [4] A. GRAVOUIL, N. MOES, and T. BELYTSCHKO. Non-planar 3D crack growth by the extended finite element and level sets - Part II : Level set update. International Journal for Numerical Methods in Engineering, 53 :2569–2586, 2002.
- [5] D. COLOMBO. An implicit geometrical approach to level sets update for 3D non planar XFEM crack propagation. submitted to Computer Methods in Applied Mechanics and Engineering, 39–50 :237–240, 2012.
- [6] S. GENIAUT, E. GALENNE, A simple method for crack growth in mixed mode with X-FEM, International Journal of Solids and Structures, Volume 49, Issue 15-16, pp. 2094–2106, 2012.

## Fracture models for elasto-plastic materials as limits of gradient damage models coupled with plasticity: the antiplane case

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We study the asymptotic behavior of a variational model for damaged elasto-plastic materials in the case of antiplane shear, based on [1]. We are interested in the case when the energy functionals depend on a small parameter  $\varepsilon$  which forces damage concentration on regions of codimension one.

More precisely, let us denote by  $\Omega \subset \mathbb{R}^n$  (n = 2 in the case of a 3D body) the reference configuration and by  $u: \Omega \to \mathbb{R}$  the scalar function that describes the displacement in the antiplane case. As usual in the theory of small strain elasto-plasticity, the displacement gradient is decomposed as  $\nabla u = e + p$ , where e and p are vector functions, representing the elastic and the plastic part of the strain, respectively. The damage variable is a scalar function  $\alpha: \Omega \to [0,1]$ , with  $\alpha = 1$  indicating the sound material and  $\alpha = 0$  corresponding to the maximum possible damage. We consider the total energy functional defined by

$$\mathcal{E}_{\varepsilon}(e, p, \alpha) := Q_{\varepsilon}(e, \alpha) + \mathcal{H}(p, \alpha) + \mathcal{W}_{\varepsilon}(\alpha),$$

where

where  $Q_{\varepsilon}(e, \alpha) := \frac{1}{2} \int_{\Omega} (\alpha + \varepsilon^2) |e|^2 dx$  is the elastic energy,  $\mathcal{H}(p, \alpha) := \int_{\Omega} \frac{\kappa(\alpha)}{\kappa(\alpha)} |p| dx$  is the energy dissipated in the plastic process, and  $\mathcal{W}_{\varepsilon}(\alpha) := \int_{\Omega} \frac{W(\alpha)}{\varepsilon} dx + \varepsilon \int_{\Omega} |\nabla \alpha|^2 dx$  is the energy dissipated by the damage process.

Here the stress constraint is given by  $|\sigma| \le \kappa(\alpha)$ , where  $\sigma = (\alpha + \varepsilon^2)e$  indicates the stress,  $\kappa: [0,1] \to \mathbb{R}$  is a nondecreasing function with  $0 < \kappa(0) < \kappa(1) < +\infty$ , while  $W: [0,1] \to \mathbb{R}$ is a continuous and decreasing function with W(1) = 0.

Since  $\mathcal{H}$  has linear growth in p, it is convenient to extend it to the space of bounded measures  $\mathcal{M}_b(\Omega;\mathbb{R}^n)$ , by setting  $\mathcal{H}(p,\alpha) := \int_{-\infty}^{\infty} \kappa(\alpha) d|p|$ , where |p| denotes the total variation of the vector measure p. This leads us to consider the displacement u in the space  $BV(\Omega)$  of functions of bounded variation in  $\Omega$ .

To describe the asymptotic behavior of  $\mathcal{E}_{\varepsilon}$  as  $\varepsilon \to 0$ , we introduce the functionals  $\mathcal{F}_{\varepsilon}: BV(\Omega) \times H^1(\Omega; [0,1]) \to [0, +\infty)$  which depend only on the displacement *u* and on the damage variable  $\alpha$  and represent the energy of the optimal additive decomposition of the displacement gradient:

$$\mathcal{F}_{\varepsilon}(u,\alpha) := \min_{e,p} \{ \mathcal{E}_{\varepsilon}(e,p,\alpha) : e \in L^{2}(\Omega;\mathbb{R}^{n}), \ p \in \mathcal{M}_{b}(\Omega;\mathbb{R}^{n}), \ Du = e+p \}.$$

Note that the minimum is achieved at a unique pair (e, p) and  $\mathcal{F}_{\varepsilon}$  can be written explicitly in an integral form. We prove the following:

**Theorem** The functionals  $\mathcal{F}_{\varepsilon}$   $\Gamma$ -converge to  $\mathcal{F}_0$  in  $L^1(\Omega) \times L^1(\Omega)$  as  $\varepsilon \to 0$  and for every  $u \in BV(\Omega)$ 

$$\mathcal{F}_0(u,1) = \min_{e,p} \left\{ \frac{1}{2} \int_{\Omega} |e|^2 \, \mathrm{d}x + \kappa(1) |p|(\Omega \setminus J_u) + \int_{J_u} \Psi(|[u]|) \, \mathrm{d}\mathcal{H}^{n-1} \right\}$$

The minimum is taken among all  $e \in L^2(\Omega; \mathbb{R}^n)$ ,  $p \in \mathcal{M}_b(\Omega; \mathbb{R}^n)$  such that Du = e + p, and it is attained at a unique pair (e, p).

The jump set  $J_u$  of u represents a crack (in the reference configuration) and the term  $\int_{J_u} \Psi(|[u]|) d\mathcal{H}^{n-1}$  can be interpreted as the energy dissipated in the process of crack open-

ing. The function  $\Psi$  is explicitly given in terms of  $\kappa$  and W and satisfies the following properties:

 $\cdot$  it is concave;

·  $\kappa(0) t \leq \Psi(t) \leq \kappa(1) t$  if t is small;

 $\Psi(t)$  is constantly equal to  $c_W = 4 \int_0^1 \sqrt{W(s)} ds$  if t overtakes a suitable threshold. Thus the crack energy depends also on the crack opening |[u]|, and the force between the crack lips (given by the derivative of  $\Psi$ ) vanishes when the crack opening is large enough. Therefore the minimization of the energy  $\mathcal{F}_0$  provides a cohesive crack model.

In the special case when  $\kappa$  is constant the limit functional describes Dugdale's cohesive model in fracture mechanics. A different approximation of this model has been recently obtained in [2]. A similar problem for elastic materials with damage and without plastic slips can be solved using the approximation results for the Mumford-Shah functional obtained by Ambrosio and Tortorelli. Those results correspond formally to our problem with  $\kappa(\alpha) = +\infty$  for every  $0 \le \alpha \le 1$ . Indeed, in this case  $|D^c u| = 0$  whenever the  $\Gamma$ -limit is finite and the term depending on  $J_u$  reduces to  $c_W \mathcal{H}^{n-1}(J_u)$ , which corresponds to Griffith's model in fracture mechanics for a brittle material with toughness  $c_W$ .

- R. Alessi, J.-J. Marigo, S. Vidoli. Gradient damage models coupled with plasticity: variational formulation and main properties. *Mechanics of Materials*, 80 (B), 351– 367, 2015.
- [2] S. Conti, M. Focardi, F. Iurlano. Phase field approximation of cohesive fracture models. Preprint 2014. http://cvgmt.sns.it/paper/2443.

## Recent progress and remaining challenges in the mathematics of sharp interface fracture evolution

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The mathematics needed to address even the most basic issues for sharp-interface fracture, such as existence of solutions, can be exceptionally technical. Yet fruitful interaction between engineers and mathematicians requires each group to have an understanding of the fundamental issues of the other group. Fortunately, the basic mathematical difficulties are actually quite intuitive. The goal of this talk is to describe these difficulties in a clear, non technical way, and also report on some recent progress.

## Representation of stress in plane cracked solids with Laurent series

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Williams series appear to be the most favored analytical tool for the description of mechanical fields near crack-tips in planar solids. Many computational methods (the Scaled Boundary Finite Element Method, the Hybrid Crack Element method, XFEM...) and experimental post-processing techniques take advantage of this approach so as to deal with the singularity at the crack-tip and the displacement discontinuity across crack-lips.

However, the series convergence domain is restricted to the interior of a disk centered at the crack-tip where the expansion is performed. The disk radius corresponds to the shortest distance between the crack-tip and the closest next singularity [1]. Therefore, Williams series is limited to represent stress and displacement in the sole crack-tip area.

In the case of a load-free double-end straight crack, it has been shown that a new kind of series expansion can be performed for the region beyond Williams series convergence disk [1]. This communication provides a generalisation of that result. It is shown that stress and displacement states can be described with Laurent series in successive annuli centered at the crack-tip. The general analytical procedure is described and several examples are provided so as to assess the validity of the approach. This technique may find applications in the aforementioned fields where Williams series are currently used.

#### References

[1] G. Hello, M. Ben Tahar, J.-M. Roelandt. Analytical determination of coefficients in crack-tip stress expansions for a finite crack in an infinite plane medium. *International Journal for Solids and Structures*, 49, 556-566, 2012.



Figure 1: Laurent series expansions with 10 (a), 20 (b) and 40 (c) terms for the fracture configuration of a thin rigid wedge inserted in a semi-infinite crack

# Mixed mode fracture separation in orthotropic media due to mechanical and thermal effects

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The crack initiation is the one of the most important factors in the collapse of mechanical and civil engineering structures as aircraft, nuclear, buildings and pavement structures [1]. In some cases, these initial cracks can be caused by a combination of mechanical and thermal fields. So, the mixed-mode fracture coupling mechanical and thermal loads in isotropic and orthotropic material like wood is investigated in this work. The analytical formulation of the energy release rate is introduced by T and A integrals that take into account mixed mode fracture, thermal process and pressure applied on the crack lips [2]. This formulation is based on Nother's theorem and the definition of the strain energy density in Lagrangian and Eulerian configurations. Moreover, this analytical formulation is implemented in finite element software Cast3m. Several numerical examples, dealing with isotropic material, are provided to evaluate the accuracy of the FEM model. For instance, the domain independency is shown in opening mode, in shear mode and in case of pressure on crack lips. Then, the crack resistance of a timber Compact Tension Specimen is investigated to show the efficiency of the proposed approach in the case of orthotropic materials.

- [1] F. Dubois, C. Petit, N. Sauvat, B. Peuchot. Diagnostic et comportement des ponts à ossature bois : Application au pont de Merle, European Journal of Civil Engineering, 10(2),191-208, 2006
- [2] R. Moutou Pitti, F. Dubois, C. Petit. Generalization of T and A integrals to timedependent materials: analytical formulations. International Journal of Fracture, 161,187-198, 2010
# Study of fracture patterns generated by different failure criteria in ordinary state-based Peridynamics

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A new theory of continuum called Peridynamics has been proposed in 2000 [1] in order to overcome the limitations of the classic theory of continuum mechanics, which is based on differential equations and consequently cannot describe material behaviour in which discontinuities, such as cracks and inclusions, are involved. Differently from the classic theory, Peridynamics is a non-local theory based on integral equations which can be applied even on discontinuous media.

Since its introduction, Peridynamics has been employed for reproducing several damage process phenomena, such as crack nucleation and branching in homogeneous materials [2] or delamination in composites [3]. Most of the articles found in literature deals with bond-based Peridynamics, which was demonstrated to be a particular case of state-based Peridynamics [4], but is characterized by some limitations on the material which can be described, for example Poisson's ratio is fixed. In state-based Peridynamics there are no limitations and material with different Poisson's ratio can be simulated. Two variants of state-based Peridynamics were introduced, the ordinary state-based, in which the nonlocal interaction force is aligned with the bond current direction, and the non-ordinary state-based, in which the force is not necessarily aligned with it.

Fundamental parameters of this theory are the size of the non-local interactions, called horizon, and how many connections, properly called bonds, have any material node in which the body is discretized. Particularly, the number of bonds is related to the so called m-ratio, which is the ratio between the horizon and the grid spacing.

In this study, some examples simulated with ordinary state-based Peridynamic 2D models are presented employing some fracture criteria in order to show how they reproduce the experimental crack patterns and directionality for brittle materials; accordingly, the influence of the m-ratio on the dynamic fracture pattern is investigated for every fracture criteria as well.

#### References

- [1] S.A. Silling. Reformulation of elasticity theory for discontinuities and long-range forces. Journal of the Mechanics and Physics of Solids, 48(1), 175-209, 2000.
- [2] Y. D. Ha, F. Bobaru. Studies of dynamic crack propagation and crack branching with peridynamics. International Journal of Fracture, 162(1-2), 229-244, 2010.
- [3] J. Xu, A. Askari, O. Weckner, S. A. Silling. Peridynamic analysis of impact damage in composite laminates. Journal of Aerospace Engineering, 21(3), 187-194, 2008.
- [4] P. Seleson, M. Parks. On the role of the influence function in the peridynamic theory. International Journal of Multiscale Computational Engineering, 9(6), 689-706, 2011.

### **Intrinsic Brittleness of Magnesium**

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The competition between dislocation emission and cleavage at a crack tip plays an important role in governing the intrinsic fracture behavior of crystalline materials. This competition is not well understood in magnesium which, due to its hexagonal close-packed structure, has many different combinations of cleavage planes and dislocation slip systems. Here, using both anisotropic linear elastic fracture mechanics theory and atomistic simulations, the emission/cleavage competition in magnesium is evaluated for a comprehensive set of crack orientations and crack tip geometries under mode I crack tip stress intensity loading at T=0K. The atomistic simulations use a new interatomic potential for Mg that accurately captures all relevant dislocation structures, unstable stacking fault energies, and surface energies, relevant to cleavage and dislocation emission. Theory predicts that the cleavage/emission competition is very close in many cases, with cleavage being favored in many crack orientations, including basal plane cracks, tensile twin and basal-prismatic plane interface cracks. Simulations reveal that nearly all cases are brittle, i.e. cleavage occurs in preference to dislocation emission. The robustness of these findings is further verified by simulations starting with an initially blunted crack tip, which does not significantly change the behavior. These results suggest that magnesium has extremely low intrinsic fracture toughness; this is consistent with recent experimental observations of many different cleavage-like planes in low-temperature fracture experiments. The occurrence of ductile-like failure at room temperature and moderate loading rates suggests that the temperature dependence of the surface and unstable stacking fault energies shifts the material toward a preference for emission and preliminary results at finite temperature are discussed. Alloying provides another avenue for changing the relevant energies in the system. Based on the T=0K properties, a more-ductile material that would be less sensitive to temperature and loading rate, would require alloying sufficient to reduce the unstable stacking fault energies for pyramidal and basal slip by  $\sim 50\%$  and  $\sim 20\%$ , respectively, relative to any associated changes in surface energy. Such large changes suggest that Magnesium and its alloys will remain susceptible to easy crack formation, lowtoughness fracture, and macroscopic brittleness in structural applications.

#### References

[1] Z. X. Wu and W. A. Curtin. Brittle and Ductile Crack-Tip Behavior in Magnesium. Acta Materialia, to appear, 2015.

## On the morphology of dynamic cracks surfaces and how to resolve them computationally

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In dynamic brittle fracture of, for example, glass plates, besides the complex failure pattern, one observes a variety of crack surface morphologies: smooth and planar, smooth and twisted, rough with sharp ridges, etc. There appears to be a correlation between the crack type and location (radial, circumferential, parallel to the boundaries of the sample, Hertzian), and the particular crack surface morphology.

In this presentation I will show experimental results from impact fragmentation on a thin glass plate and discuss a peridynamic model and results that mimic the conditions in the experiment ([1]). The combination of "clean fracture" and damage and fragmentation makes it difficult to simulate this type of dynamic brittle fracture/fragmentation with methods that try to track individual cracks. This is one reason for using peridynamics with a meshfree discretization in this type of problems. Our computational investigation reveals the reasons behind the different crack surface morphologies: elastic waves propagating, reflecting from the plate boundaries, and interacting/reinforcing each other are responsible for the observed behavior. This study answers the question whether the crack patterns and crack surface morphologies observed in fracture and fragmentation of a glass plate are controlled by the microstructural details of glass or by the dynamics of fracture in a homogeneous and isotropic material. The advantage of using the nonlocal damage introduced by the peridynamic approach rests in its ability of allowing the initiation, growth, and propagation of cracks autonomously and without having to explicitly track them ([2]).

#### References

[1] W. Hu, Y. Wang, J. Yu, C.F. Yen, F. Bobaru. Impact damage on a thin glass plate with a thin polycarbonate backing. *International Journal of Impact Engineering*, 62, 152-165, 2013.

[2] F. Bobaru, Y.D. Ha, W. Hu. Damage progression from impact in layered glass modeled with peridynamics. *Open Engineering*, 2(4), 551-561, 2012.

## Effect of brittle fracture in a metaconcrete slab under shock loading

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We have proposed a new type of concrete for the attenuation of shock waves induced by dynamic excitation, which we have named *metaconcrete*. Inspired by the metamaterials used for the manipulation of electromagnetic and acoustic waves, this new metamaterial for the mitigation of shock waves utilizes the activation of resonance within specially engineered inclusions. Metaconcrete replaces the standard stone and gravel aggregates of regular concrete with spherical inclusions consisting of a heavy core coated with a compliant outer layer. Finite element studies of this new metamaterial using purely elastic constituents reveal trapping of the supplied energy within the inclusions and a reduction in mortar stress [1]. To further investigate this effect we now consider the brittle behavior of mortar by incorporating the fracture characteristics of the mortar matrix through the use of an eigenerosion scheme.

Fracture in mortar can be conveniently modelled through the use of an eigenerosion algorithm [2], derived from the more general eigenfracture approach [3]. Eigenfracture is an approximation scheme for variational models of Griffith's theory of fracture [4, 5] that resorts to the classical device of eigendeformations [6] in order to account for material fracture. To this end, the energy functional of the system is assumed to be dependent on two fields: the displacement field and an eigendeformation field, that describes cracks as may be present in the body.

In the eigenfracture scheme the fracture energy is set to be proportional to the volume of the  $\varepsilon$ -neighborhood of the support of the eigendeformation field, suitably scaled by  $1/\varepsilon$ . The optimal crack set is obtained by minimizing the energy functional with respect to both the displacement and the eigendeformation fields, subject to irreversibility constraints. Eigenerosion is derived from the general eigenfracture scheme by restricting the eigendeformations to be either zero, in which case the local behavior is elastic, or equal to the local displacement gradient, in which case the corresponding material neighborhood is eroded. When combined with spatial discretization, this scheme gives rise to element erosion, i.e., each element can be either intact, in which case its behavior is elastic, or be eroded and has no load bearing capacity. The convergence properties of the eigenerosion scheme for mode I fracture propagation in three dimensional problems have been discussed in [2].

To investigate the effect of brittle fracture using eigenerosion within the mortar phase of metaconcrete we consider a rectangular section of an infinite metaconcrete slab with a periodic array of aggregates, the same as that used in the elastic simulations of [1]. Each aggregate has a 12mm outer radius and consists of a lead core coated with a 1-3mm compliant material that allows for oscillation, and thus resonance, of the heavy lead core. The slab section contains 8 whole spherical inclusions and 28 quarter spheres surrounding the outer edges. The system is excited by an explosion located at a small distance from the end face that causes the initiation of a shock wave within the slab.

By comparing the resulting wave propagation profiles for both metaconcrete and homogeneous slabs, we can observe the changes in behavior due to the of the presence of layered aggregates. For each case we see a similar amount of damage at the explosion site, however, observe a reduction in the stress intensity and shock velocity in the metaconcrete slab with comparison to the homogeneous slab. The shock front is delayed and trapped within the metaconcrete aggregates, whereas the homogeneous slab, by constrast, displays a planar shock front that progresses more quickly through the length of the slab. A similar result was seen in the elastic analyses. With the inclusion of fracture properties in the model, these results provide a more accurate understanding of metaconcrete behavior under dynamic loading and suggest that enhanced performance may be gained by utilizing a metaconcrete slab for shock wave mitigation and blast shielding applications. The investigation into different aggregate material properties, geometry, mortar strength, and explosion magnitude allows us to fully characterize metaconcrete behavior under blast wave loading and therefore understand the best configuration for different applications.

### References

- S. J. Mitchell, A. Pandolfi, M. Ortiz. Metaconcrete: designed aggregates to enhance dynamic performance. *Journal of the Mechanics and Physics of Solids*, 65:69–81, 2014.
- [2] A. Pandolfi and M. Ortiz An eigenerosion approach to brittle fracture. *International Journal for Numerical Methods in Engineering*, 92:694–714, 2012.
- [3] B. Schmidt, F. Fraternali, and M. Ortiz. Eigenfracture: an eigendeformation approach to variational fracture. *SIAM Multiscale Modeling and Simulation*, 7:1237–1266, 2009.
- [4] L. Ambrosio and V. M. Tortorelli. On the approximation of free discontinuity problems. *Bollettino dell'Unione Matematica Italiana B*, 7:105–123, 1992.
- [5] G. A. Francfort and J. J. Marigo. Revisiting brittle fracture as an energy minimization problem. *Journal of the Mechanics and Physics of Solids*, 46:1319-1342, 1998.
- [6] G. Colonnetti. Su certi stati di coazione elastica che non dipendono da azioni esterne. *Rendiconti della R. Accademia dei Lincei, Giugno*, 43–47, 1917.

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